On the Impact of using Mixed Integer Programming Techniques on Real-world Offshore Wind Parks

Martina Fischetti[1,2] and David Pisinger[2]
[1]Vattenfall BA Wind, Jupitervej 6, 6000 Kolding, Denmark
[2]Technical University of Denmark, Operations Research, DTU Management, Produktionstorvet, 424 DK-2800 Kgs. Lyngby, Denmark

Keywords: Optimization, Mixed Integer Linear Programming, Wind Energy, Routing, Offshore Cables, Green Energy.

Abstract: Wind power is a leading technology in the transition to sustainable energy. Being a new and still more competitive field, it is of major interest to investigate new techniques to solve the design challenges involved. In this paper, we consider optimization of the inter-array cable routing for offshore wind farms, taking power losses into account. Since energy losses in a cable depend on the load (i.e. wind), cable losses are estimated by considering a possibly large number wind scenarios. In order to deal with different wind scenarios efficiently we used a precomputing strategy. The resulting optimization problem considers two objectives: minimizing immediate costs (CAPEX) and minimizing costs due to power losses. This makes it possible to perform various what-if analyses to evaluate the impact of different preferences to CAPEX versus reduction of power losses. Thanks to the close collaboration with a leading energy company, we have been able to report results on a set of real-world instances, based on six existing wind parks, studying the economical impact of considering power losses in the cable routing design phase.

1 INTRODUCTION

With a total global capacity of more than 400 GW by the end of 2015, wind power is a leading technology in the transition away from fossil fuels. Having a yearly market growth of 15-20%, it is however necessary to face new challenges on a market that is always more competitive. According to [Gonzlez et al., 2014] the expenses for electrical infrastructure of a offshore wind farm account for 15-30% of the overall initial costs. Therefore, high-level optimization in this area is a key factor. In this work we focus on the cable routing between offshore wind turbines (the so called inter-array optimization).

The power production of offshore turbines is collected through one or more substations and then conveyed to the coast. The cabling will therefore constitute a tree of cables from each substation to the connected turbines. Different cables with different costs, capacities and resistances are available on the market and the task is therefore not only to connect the turbines in the cheapest possible way, but also to choose appropriate dimensions of the cables to minimize losses.

Thanks to the collaboration with a leading energy company it has been possible to build a detailed model including nearly all the constraints arising in practical applications, and to evaluate the savings of optimized layouts on real cases. The resulting optimization tool has been validated by company experts, and is now routinely used by the planners.

Wind park cable routing optimization has obtained considerable attention in the last years. Due to the large number of constraints and the intrinsic complexity of the problem, many studies (i.e. [Dutta and Overbye, 2011, Gonzlez-Longatt and Wall, 2012, Li et al., 2008, Zhao et al., 2009]) preferred to use adhoc heuristics. Only a few papers used Mixed Integer Linear Programming (MILP), notably [Bauer and Lysgaard, 2015, Fagerfjall, 2010, Dutta, 2012, Berzan et al., 2011,Hertz et al., 2012,Cerveira et al., 2016,Pillai et al., 2015]. A MILP approach boosted with heuristics (a so-called mat-heuristic approach) to deal with large-scale wind parks in an acceptable time has been recently proposed in [Fischetti and Pisinger, 2016]. The present work is based on [Fischetti and Pisinger, 2016] but focuses more on real applications of the optimization model and on its economical impact. Several variants of the problem have been proposed in the literature. To the best of our knowledge

Fischetti M. and Pisinger D.
On the Impact of using Mixed Integer Programming Techniques on Real-world Offshore Wind Parks.
DOI: 10.5220/0006190701080118
In *Proceedings of the 6th International Conference on Operations Research and Enterprise Systems (ICORES 2017)*, pages 108-118
ISBN: 978-989-758-218-9

only [Cerveira et al., 2016] has considered power loss in cables. However, [Cerveira et al., 2016] does not take into account variable cable loads due to fluctuating wind. [Bauer and Lysgaard, 2015] proposes an Open Vehicle Routing approach for this problem adding the planarity constraints on the fly. In this Open Vehicle Routing version of the problem, only one cable can enter a turbine, even if this is often not the case in the reality. In [Bauer and Lysgaard, 2015], the possibility of branching cables in the turbines (as we are doing), is mentioned as a future work. However, the substation limits, that could be a major constraint in practical applications, are not considered in [Bauer and Lysgaard, 2015]. Different approaches for the cable network design are provided in [Berzan et al., 2011]. The suggested approach is a divide-and-conquer heuristic based on the idea of dividing the big circuit problem into smaller circuit problems. The proposed MILP model cannot deal with more than 11 turbines. In [Hertz et al., 2012] the cable layout problem for onshore cases is studied. The onshore cable problem is similar to the offshore one with the following differences. First of all, the cable can be of two types: underground cables (connecting turbines to other turbines or to the above-ground level), and above-ground cables. In the first case, the cables need to be dug in the ground. Due to the fact that parallel lines can use the same dug hole, parallel structures are preferred (until a fixed number). The above-ground level cables need to follow existing roads. Such constraints do not exist in the offshore case.

The main contribution of this paper is to analyse how the inter-array cable routing of real-world wind farms can be improved by using modern optimization techniques. A particularly challenging aspect in the cable routing design, is to understand if one could limit power losses by optimizing cable routing. As a general rule, cables with less resistance are also more expensive, therefore we would like here to make a proper trade-off between investments and cable losses. We formulate the optimization problem with immediate costs (CAPEX) and losses-related costs as two separate goals. The two objectives can be merged into a single objective by proper weighing of the two parts. The weighing factor can be considered fixed or can vary: this makes it possible to perform various what-if analyses to evaluate the impact of different preferences (i.e. of different weighing factors). The latter approach is important in cases where a positive pay-back is demanded within a short time horizon, or where liquidity problems hinder choosing the best long-term solution. We report a study of both approaches on a set of real-world instances.

In order to perform the above analysis, we developed a MIP approach to optimize the routing. In the computation of power losses, it is shown that wind scenarios can be handled efficiently as part of data preprocessing, resulting in a MIP model of tractable size. Tests on a library of real-life instances proved that substantial savings can be achieved.

The rest of the paper is organized as follows: Section 2 describes our MILP model, first presenting a basic model and then improving and extending the formulation. In particular, we show how to model power losses, and propose a precomputing strategy that is able to handle this non-linearity efficiently, thus avoiding sophisticated quadratic models that would make our approach impractical. Section 3 compares our optimized solutions with an existing cable layout for a real wind farm (Horns Rev 1), showing that millions of euro can be saved. Section 4 is dedicated to various what-if analyses. Subsection 4.1 describes the real-world wind farms that we considered in our tests while Subsection 4.2 shows the results of our optimization on a testbed of real-world cases, reporting the impact of considering power losses for all the instances. Subsection 4.3 is dedicated to the Pareto optimality study. Some conclusions are finally addressed in Section 5.

2 MATHEMATICAL MODEL FOR CABLE ROUTING OPTIMIZATION

2.1 Basic Model

In the present paper we assume that the location of the turbines has already been defined. We wish to find an optimal cable connection between all turbines and the given substation(s), minimizing the total cable costs. The optimization problem considers that:

- the energy flow leaving a turbine must be supported by a single cable;
- the maximum energy flow (when all the turbines produce their maximum) in each connection cannot exceed the capacity of the installed cable;
- different cables, with different capacities, costs and impedances, can be installed;
- cable crossing should be avoided;
- a given maximum number of cables can be connected to each substation;
- cable losses (dependent on the cable type, the cable length and the current flow through the cable) must be considered in the optimization.

We will first model the problem without cable losses and then discuss in Subsection 2.2 how to efficiently express these constraints. We model turbine positions as nodes of a complete and loop-free directed graph $G = (V,A)$ and all possible connections between them as directed arcs. Some nodes correspond to the substations that are considered as the roots of the trees, being the only nodes that collect energy. Let P_h be the power production at node h. We distinguish between two different types of node:

$$h \in \begin{cases} V_T & \text{if the } h\text{-th node correspond to a turbine} \\ V_0 & \text{if the } h\text{-th node correspond to a substation} \end{cases}$$

Let T denote the set of different cable types that can be used. Each cable type t has a given capacity k_t and unit cost u_t, representing the cost per meter of the cable (CAPEX). Arc costs can therefore be defined as $c^t_{i,j} = dist(i,j)u_t$ for each arc (i,j) and for each type $t \in T$, where $dist(i,j)$ is the distance between turbine i and turbine j. In our model we use the continuous variables $f_{i,j} \geq 0$ for the maximum flow on arc (i,j). The binary variables $x^t_{i,j}$ define cable connections as

$$x^t_{i,j} = \begin{cases} 1 & \text{if arc } (i,j) \text{ with cable type } t \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

Finally, variables $y_{i,j}$ indicate whether turbines i and j are connected (with any type of cable). Note that variables $y_{i,j}$ are related to variables $x^t_{i,j}$ as $\sum_{t\in T} x^t_{i,j} = y_{i,j}$. The overall model becomes:

$$\begin{array}{llll}
\min & \sum_{i,j\in V} \sum_{t\in T} c^t_{i,j} x^t_{i,j} & & (1) \\
\text{s.t.} & \sum_{t\in T} x^t_{i,j} = y_{i,j} & i,j \in V : j \neq i & (2) \\
& \sum_{i:i\neq h} (f_{h,i} - f_{i,h}) = P_h & h \in V_T & (3) \\
& \sum_{t\in T} k_t x^t_{i,j} \geq f_{i,j} & i,j \in V : j \neq i, & (4) \\
& \sum_{j:j\neq h} y_{h,j} = 1 & h \in V_T & (5) \\
& \sum_{j:j\neq h} y_{h,j} = 0 & h \in V_0 & (6) \\
& \sum_{i\neq h} y_{i,h} \leq C & h \in V_0 & (7) \\
& x^t_{i,j} \in \{0,1\} & i,j \in V, t \in T & (8) \\
& y_{i,j} \in \{0,1\} & i,j \in V & (9) \\
& f_{i,j} \geq 0 & i,j \in V, j \neq i & (10)
\end{array}$$

The objective function (1) minimizes the total cable layout cost. Constraints (2) impose that only one type of cable can be selected for each built arc, and defines the $y_{i,j}$ variables. Constraints (3) are flow conservation constraints: the energy (flow) exiting each node h is equal to the flow entering h plus the power production of that node (except if the node is a substation). Constraints (4) ensure that the flow does not exceed the capacity of the installed cable, while constraints (5) and (6) impose that only one cable can exit a turbine and none can exit the substations (tree structure with root in the substations). Finally, constraints (7) impose the maximum number of cables (C) that can enter each substation.

In order to model no-crossing constraints we need a constraint for each pair of crossings arcs, i.e. a huge number of constraints. We have, therefore, decided to generate them on the fly, as also suggested in [Bauer and Lysgaard, 2015]. In other words, the optimizer considers model (1) - (10) and adds the following new constraints whenever two established connections (i,j) and (h,k) cross

$$y_{i,j} + y_{j,i} + y_{h,k} + y_{k,h} \leq 1. \qquad (11)$$

The reader is referred to [Fischetti and Pisinger, 2016] for stronger versions of those constraints. Using this approach, the number of non-crossing constraints actually added to the model decreases dramatically, making the model faster to solve. As presented, the model is able to deal with small size instances only. In order to produce high quality solutions in an acceptable amount of time also for big instances a "matheuristic" framework (as the one proposed in [Fischetti and Pisinger, 2016]) should be used on top of this basic model.

2.2 Cable Losses

In this section we propose an extension of the previous model taking cable losses into account. Let us consider a generic cable (i,j) of type t, supporting a current $g^t_{i,j}$. Power losses increase with the square of the current, according to the formula:

$$3R^t \cdot dist(i,j)(g^t_{i,j})^2 \qquad (12)$$

where R^t is the electrical resistance of the 3-phase cable of type t, in Ω/m. The current $g^t_{i,j}$ obviously depends on the considered wind scenario. As a consequence, dealing with equation (12) directly in the model, would imply dealing with non-linearities over multiple scenarios. Nevertheless, (12) can be simplified if we assume that all the turbines in the park have the same power production under the same wind scenario. This is a fair assumptions since typical parks are constructed by using only one turbine model and wake effect is not usually considered in electrical studies. Under this assumption, the current I_s passing through a generic cable (i,j) of type t under scenario s, can be expressed as a function of the number f of turbines supported by the cable as:

$$PLoss^{t,f,s} = (fI^s)^2 R^t dist(i,j). \qquad (13)$$

The value $f = 1,...F$ is limited by the capacity of the cables. By introducing the dependency on f in our main binary variables (now $x_{i,j}^{t,f}$) we can re-write our two objectives as:

$$min \textstyle\sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} \sum_{s \in S} \pi_s PLoss^{t,f,s} x_{i,j}^{t,f} \quad (14)$$

and

$$\min \quad \textstyle\sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} c_{i,j}^{t} x_{i,j}^{t,f} \quad (15)$$

where π_s is the probability of scenario s. As we have discussed earlier, minimizing losses can imply an increase of the CAPEX cost, therefore the two objective must be properly balanced. In some cases (e.g., when there is no limit on the CAPEX) they can be merged, by using a converting factor for the loss-related term: this is the estimated cost for each MW of production lost over the wind farm lifetime (Net Present Value). This value (denoted as K) is an input value, that the designer can set to the desired project-specific value. The merged objective function, now expressed in €, is then:

$$\min \sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} c_{i,j}^{t} x_{i,j}^{t,f}$$
$$+K \sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} \sum_{s \in S} \pi_s PLoss^{t,f,s} x_{i,j}^{t,f} \quad (16)$$

We notice that (16) can be rewritten as:

$$\min \sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} u_t dist(i,j) x_{i,j}^{t,f}$$
$$+K \sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} \sum_{s \in S} 3\pi_s (fI^s)^2 R^t dist(i,j) x_{i,j}^{t,f} =$$
$$\min \sum_{i,j \in V} \sum_{t \in T} \sum_{f \in F} (u_t + K \sum_{s \in S} 3\pi_s (fI^s)^2 R^t) dist(i,j) x_{i,j}^{t,f} \quad (17)$$

The non-linear expressions in the objective function (17) can actually be handled implicitly in a pre-processing phase, without changing the original model (1)-(10) at all, according to the following idea. We consider the basic model (1)–(10) without cable losses on a modified instance where each cable type is replaced by a series of "subcables" with discretized capacity and modified cable cost taking both CAPEX and revenue losses due to cable losses into account.

Nearly all wind farms are designed for only one turbine type, hence the maximum power production P_h of each turbine can be assumed to be 1, meaning that we can express the cable capacity as the maximum number of turbines supported. Consider a certain cable type t that can support up to k_t turbines. We replace it by k_t "subcable" types of capacity f = 1, 2, ..., k_t whose unit cost is computed by adding both cable/installation unit costs (u_t) and loss costs (denoted as $loss^{t,f}$) considering the current produced by exactly f turbines. Note that such unit costs increase with f, so the optimal solution will always select the subcable type f supporting exactly the number of turbines connected, hence the approach is correct.

The above approach allows us to easily consider multiple wind scenarios without affecting the model size. This is obtained by precomputing the subcable costs by just considering a weighted average of the loss cost under different wind scenarios (and hence different current productions). To be more specific, if we look again at formula (17), we can now precompute the value

$$loss^{t,f} = K \textstyle\sum_{s \in S} 3\pi_s (fI^s)^2 R^t \quad (18)$$

where π_s is the probability of scenario s and I^s is the current produced by a single turbine under wind scenario s, assuming negligible wake effect, i.e., all turbines are producing the same amount of energy under a given wind scenario s. We refer to the next subsection for a more detailed example of how cable costs are pre-processed when considering losses. As said, K is a factor to estimate the value (in €) of the MW loss, and can be computed as $K = K_{euro} \cdot 8760$ where K_{euro} is the NPV for a MW/h production over the park lifetime, and 8760 is the number of hours in a year. Notice that K_{euro} acts as a weighting factor between the two objectives: minimize CAPEX costs versus minimize losses.

2.3 Loss Pre-computation

In this section we elaborate on our pre-computing strategy proposed in the previous session, using a concrete example from the real wind park Horns Rev 1. The park consists of 80 2MW turbines and is located about 15 km from the Danish shore. This park will be used as one of our test cases both in Section 3 and 4.

Fixed the turbine layout, one could consider different sets of cables to be used. Different sets can differ in cable cross section or in voltage (33kV or 66kV generally), which reflects in different capacities and resistances. The set of most adequate cables is selected by the electrical specialists in the company. Of course, different cables can lead to different solutions, as we will see in Section 4.

We will now focus on one cable set only, in order to better illustrate how different wind scenarios are handled in the pre-processing phase. Changing the cable data, the process is analogous.

Let us suppose that we are given a set of two cables: the cheapest one can support ten 2MW turbines and the most expensive fourteen turbines. This set of cables will be indicated as cb05 in Section 4. We are provided with the following table, that reports the characteristics of the two cable types.

If we want to optimize on CAPEX costs only, we just need to input to the model the capacity of each

Table 1: Cable information for cb05.

cables	type	n. of 2MW turb.	resistance [Ohm/km]	price [€/m]	install. price [€/m]
cb05	1	10	0.13	180	260
	2	14	0.04	360	260

cable type and its overall cost (cable price plus installation cost). In this case, for example, this would be:

- type 1: supports up to 10 turbines with a unit cost of 440 €/m
- type 2: supports up to 14 turbines with a unit cost of 620 €/m

Table 2 shows how the model will compute the unit price (CAPEX only) depending on the number of turbines connected.

Table 2: CAPEX costs for cb05, depending on the number of turbines connected.

cable type	n. of 2MW turb. supported	price [€/m]
1	1	440
	2	440
	3	440
	4	440
	5	440
	6	440
	7	440
	8	440
	9	440
	10	440
2	11	620
	12	620
	13	620
	14	620

Notice that, considering CAPEX costs only, the cost to use one type of cable is independent of how many turbines it is connected to (up to the capacity limit).

Let us now consider the losses in our optimization using the strategy of Subsection 2.2. As we discussed earlier, the power loss in a cable depends on the current passing through it. Since only a discrete number of turbines can be connected to each cable path, we can express the current as a function of the number f of turbines connected (as shown equation (18)) without any loss of precision in the result.

Still referring to equation (18), the losses depend also on the wind statistics in the site. We can define a wind scenario (s) as a wind speed and its probability to occur (π_s). At a given wind speed, a given turbine will produce a specific current (I_s).

Wind scenarios can be defined in different ways. In this paper we used both real measurements and scenarios derived from Weibull distributions for the specific sites. For the Horns Rev 1 case we are considering, we had real measurements from the site, i.e., a wind speed sample each 10 minutes for 10 years. We grouped all these samples in wind-speed bins of 1m/s, obtaining 25 wind scenarios (from 1 m/s to 25 m/s). The probability of each scenario was obtained looking at the frequency of the specific wind speed over all the samples. In our tests we decided to bin our data every 1 m/s, following the practice in electrical losses computations. However this should not be considered a limit: since the wind scenarios are handled in the pre-processing phase, the number of scenarios does not affect the size of the final optimization model.

Having computed I_s and π_s according to the scenario definition, power losses can now be calculated. Parameter $K_{euro} = 690$ €/MWh was computed by the company experts for a wind park lifetime of 25 years, while resistance R_t is defined according to Table 1. Using equation (18), power loss costs $loss^{t,f}$ can be now precomputed. As shown in (17), the cost considered in the objective for each cable connection will need to include the CAPEX costs (u_t) and the contribution from losses ($loss^{t,f}$). Therefore the final imput to the optimization tool for Horns Rev 1 with cb05, will be as shown in Table 3.

Table 3: Precomputed cable prices for cable cb05 (including installation costs) precomputed considering fixed costs and power losses for Horns Rev 1.

cable type	n. of 2MW turb. supported	price [€/m]
1	1	441.16
	2	442.71
	3	445.27
	4	448.87
	5	453.50
	6	459.15
	7	465.83
	8	473.54
	9	482.28
	10	492.04
2	11	639.77
	12	643.41
	13	647.36
	14	651.63

A comparison between Tables 2 and 3 shows the impact of considering losses on cable prices. While from a installation perspective the cost for each cable type is fixed, it now varies depending on how many turbines are connected. As we will see, this can have an impact on the optimal cable routing.

3 COMPARISON WITH AN EXISTING LAYOUT

We report in this section a comparison between our optimized solutions (considering and not considering losses) and the existing cable routing for Horns Rev 1, a real-world offshore park located in Denmark. Figure 1 shows the actual design for Horns Rev 1 (from [Kristoffersen and Christiansen, 2003]).

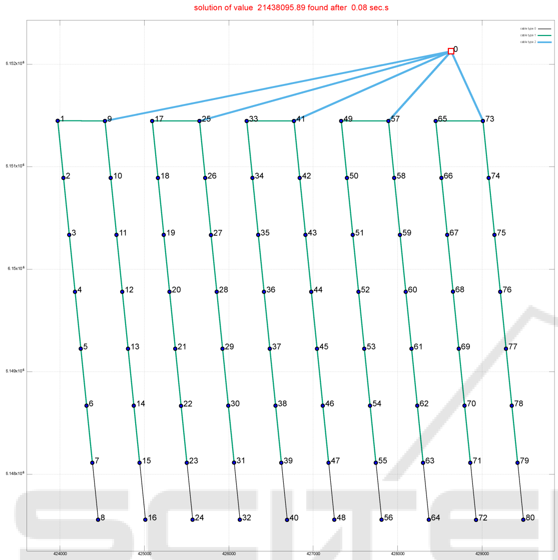

Figure 1: Existing cable routing for Horns Rev 1.

Three different types of cables are used: the thinnest cable supports one turbine only, the medium supports 8 turbines, and the thickest 16. We estimated the costs and resistances of these cables based on the cable cross section. The estimated prices are 85 €/m, 125 €/m and 240 €/m, respectively, plus an estimated 260€/m of installation costs (independent of the cable type). We ran our CAPEX optimization with the above prices obtaining the layout in Figure 2. The optimized layout is significantly different from the existing one. Looking at immediate costs, the optimized layout is over 1.5 M€ less expensive. As already said, this layout is optimized only on immediate costs, nevertheless if we estimate its value in 25 years (considering losses) this layout is still more profitable than the existing one: considering both CAPEX and losses the optimized layout is 1.6 M€ more profitable than the existing one (Net Present Value) .

By optimizing cable losses, one can further increase the value in the long term. Figure 3 shows the optimized solution considering losses (thus optimizing the value of the cable route in its lifetime). Compared with the existing layout (Figure 1), this new layout is about 1.7 M€ (NPV) more profitable in 25years, and still around 1.5 M€ cheaper at construction time.

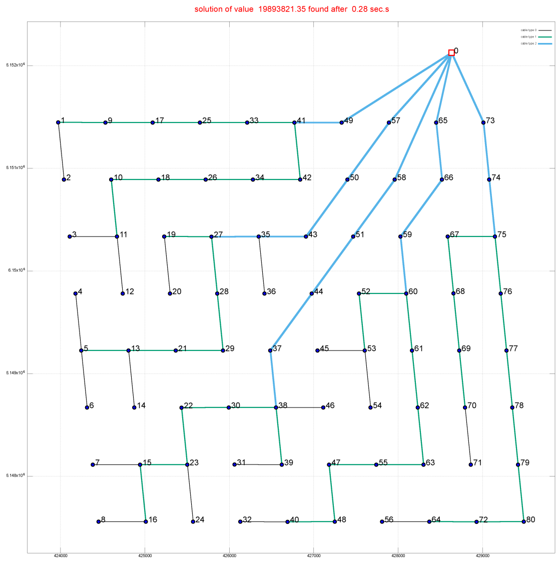

Figure 2: Optimized layout for Horns Rev 1 (CAPEX costs only): this layout results more than 1.5M € more profitable than the existing one.

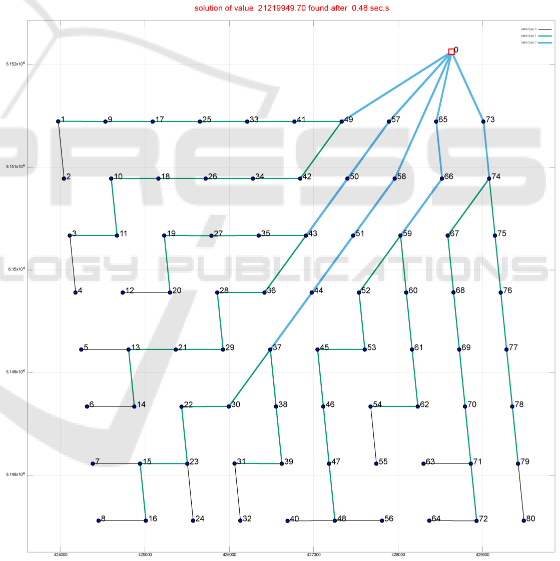

Figure 3: Optimized layout for Horns Rev 1 (considering losses): in the wind park lifetime this layout is estimated to be more than 1.7M€ more profitable than the existing one.

Table 4 summarizes the savings of the two optimized layouts compared with the existing one, both from an immediate cost perspective and from a long-term perspective: values are expressed in K€.

The test shows that millions of euros can be saved using our optimization methods on real parks. In the next section we want to focus on the other great advantage of using automatic optimization tools: the possibility of performing a number of what-if analyses. To the best of our knowledge, this is the first de-

Table 4: Savings of optimized solutions compared with the existing cable routing for Horns Rev 1.

	Savings [K€]	
opt mode	immediate	in 25years
CAPEX	1544	1605
lifetime	1511	1687

tailed study on the impact of different design choices on the cable routing itself and on its impact on immediate costs (CAPEX) and long term costs.

4 WHAT-IF ANALYSIS

We performed a number of what-if analyses on different real-world wind farms. In particular we were interest in understanding the impact of considering power losses in the design phase. We will first compare solutions optimized only for CAPEX costs, with solutions optimized looking at the whole lifetime of the park. We will then study the usage of different types of cable (with different resistances) in both cases, and the long-term savings compared with the possible higher investments costs. We will also perform a multi-criteria analysis where the user can balance between initial costs and long-term savings: this could be of interest, for example, when the company requests that the higher investment must be paid off in a limited number of years.

4.1 Test Instances

We tested our model on the real-world instances proposed in [Fischetti and Pisinger, 2016]. They consider five different real wind farms in operation in United Kingdom and Denmark, and one new wind farm under construction. These parks are Horns Rev 1, Kentish Flats, Ormonde, Dan Tysk, Thanet and Horns Rev 3.

This dataset includes old and new parks, with different power ratings and different number of turbines installed, and therefore represents a good benchmark for our tests. Each park has one substation with its own maximum number of connections (C).

In details:

- Horns Rev 1 has 80 turbines Vestas 80-2MW and $C = 10$
- Kentish Flats has 30 turbines Vestas 90-3MW. It is a near-shore wind farm, so it is connected to the onshore electrical grid without any offshore substation. Nevertheless, only one export cable is connected to the shore, therefore the starting point of the export cable is treated as a substation. We set $C = \infty$ as there is no physical substation limitation in this case.
- Ormonde has 30 Senvion 5MW and $C = 4$
- DanTysk has 80 Siemens 3.6MW and $C = 10$
- Thanet has 100 Vestas 90-3MW and $C = 10$
- Horns Rev 3 has 50 Vestas 164-8MW and $C = 12$

The dataset also includes different sets of cables, indicated as cb01, cb02, cb03, cb04 and cb05.

The cost of the cables considering power losses has been precomputed following the strategy proposed in Subsection 2.2. We computed the cable-loss prices using real measured data (for Horns Rev 1 and 3, Ormonde and DanTysk) and estimations based on Weibull distributions (Kantish Flats and Thanet).

Each combination of site (i.e. wind farm) and feasible cable set represents an instance in the testbed.

4.2 Impact of Considering Power Losses

The aim of this section is to analyse how cable routing changes when cable losses are taken into account. We used the real-world instances presented in the previous section to perform our tests. We ran our optimization tool with a time-limit of 10 hours (Intel Xeon CPU X5550 at 2.67GHz, using Cplex 12.6) in order to have high quality solutions (for the small instances these are the proven optimal solutions).

In all our instances thicker cables are more expensive and have lower resistance. This means that if the designer of the cable routing aims only at minimizing the initial costs (CAPEX), then he/she would go for the cheapest cables satisfying the load, and increase the power losses. On the contrary, focusing only on minimizing the losses, one would prefer to increase the initial costs. Using the methods explained in Section 2.2 we aim at finding the optimal balance between the two objectives, looking at the overall costs in the life time of the park.

As it can be seen from Table 5, the amount of savings varies from instance to instance, depending on the prices, on the restrictions of the specific wind farm and on the structure of the layout.

It should be noticed that the layout optimized on the lifetime always provides some savings in the long term, but the amount highly varies from case to case. In Figure 4 the case of Horns Rev 3 with cable set cb04 is shown.[1] It is seen that both the structure of the cable routing and the usage of thicker cables (green in the figure) increases in the loss-optimized layout.

---

[1] This is a preliminary layout from Vattenfall, not necessarily reflecting the final layout.

Table 5: Increase in the initial investment and long term savings for our test instances (Net Present Value). The first two columns denote the wind farm and possible cable types. The next column shows how much the investment is increased in the layout taking cable losses into account. In all test cases this amount is paid back in 25 years, and the additional savings by using the lifetime-optimized cable layout are shown in the last column.

wind farm	cable set	increase in initial investment [K€]	savings in 25y [K€]
Horns Rev 1	cb01	1	23
	cb02	24	60
	cb05	103	56
Kentish Flats	cb01	2	3
	cb02	1	4
	cb04	19	8
	cb05	5	1
Ormonde	cb03	9	0
	cb04	19	16
DanTysk	cb01	115	21
Thanet	cb04	15	92
	cb05	1	19
Horns Rev 3	cb04	42	172
	cb05	682	208

In this case the loss-optimized layout is 41 K€ more expensive at construction time (with respect to the CAPEX optimized layout). Nevertheless, in 25 years, this amount is paid back and another 172 K€ are saved (NPV).

We now try to investigate how the optimizer is restructuring the layout in order to have savings in the long run. As already noticed, every wind farm is different, so one cannot define a rule of thumb to design a good cable routing. Nevertheless, observing our layouts, we noticed a different proportion in the usage of the cable types (black and green in the figures). In particular, all the CAPEX solutions minimize the use of the expensive cables: looking only at the immediate costs, it is always preferable to go for the cheapest cable when possible, even creating longer connections. When optimizing considering losses, instead, cables with less resistance become more appealing, even if they are more expensive. In the Horns Rev 1 instance, for example, going from CAPEX optimized to lifetime-optimized the usage of type 1 cables decreases (from 55.5% of the total length to 40.3%) and the usage of type 2 cables increases (from 44.5 to 59.7%).

In Table 6 we report the cable usage (percentage of the total cable length) for all our test-bed solutions.

All in all, it can be observed from our results on real-world instances that in most cases it is convenient to invest in cables with lower resistance. The cable route and the type of cable selection for each connection is not an obvious choice and an optimization tool is necessary to determine it.

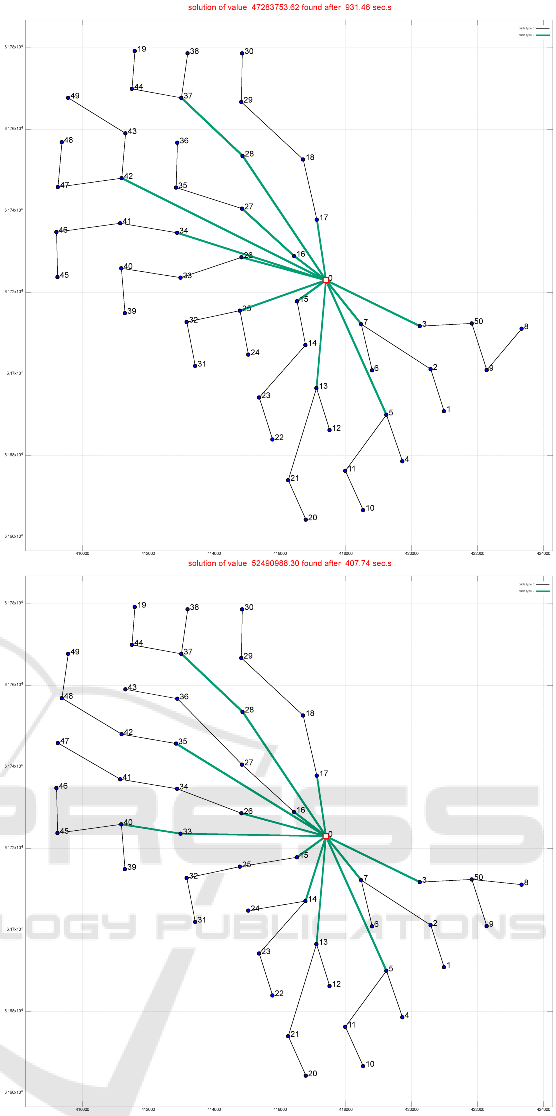

Figure 4: Optimized cable routing for Horns Rev 3, using cable set cb04. We imposed that cable type 2 can support 5 turbines only twice. The top layout is optimized only on CAPEX, the second is also considering power losses.

4.3 Bi-objectivity Tests

As discussed in Subsection 2.2, our problem has to balance between two opposite objectives: minimizing immediate costs and minimizing revenue losses in the long run. As we have seen in the previous tests, these two objectives are not always aligned since the more expensive cables have lower resistances (so less losses). The balancing factor between the two objectives is K_{euro}, that represents the price of energy (Net Present Value). Setting K_{euro} to zero, for example, means that there is no revenue from selling energy, therefore it does not matter to have losses, but it is

Table 6: Analysis on the usage of different types of cables when optimizing considering or not considering losses. The last three columns report the usage of the different cable types as percentage of the total cable length of that layout.

ID	wind farm	cable set	opt mode	length per cable type [%]		
				Type 1	Type 2	Type 3
1	Horns Rev 1	cb01	capex	55.1	40.1	4.8
2			lifetime	53.6	41.7	4.7
3		cb02	capex	57.4	42.6	
4			lifetime	44.1	55.9	
5		cb05	capex	100.0	0.0	
6			lifetime	87.7	12.3	
7	Kentish Flats	cb01	capex	66.4	33.6	0.0
8			lifetime	66.1	33.9	0.0
9		cb02	capex	66.4	33.6	
10			lifetime	60.8	39.2	
12		cb04	capex	90.1	9.9	
13			lifetime	90.1	9.9	
14		cb05	capex	95.6	4.4	
15			lifetime	95.6	4.4	
16	Ormonde	cb03	capex	69.6	30.4	
17			lifetime	76.7	23.3	
18		cb04	capex	66.9	33.1	
19			lifetime	67.4	32.6	
20	DanTysk	cb01	capex	39.0	19.4	41.7
21			lifetime	38.7	22.5	38.8
26	Thanet	cb04	capex	86.3	13.7	
27			lifetime	82.7	17.3	
28		cb05	capex	71.9	28.1	
29			lifetime	71.9	28.1	
30	Horns Rev 3	cb04	capex	57.4	42.6	
31			lifetime	60.7	39.3	
32		cb05	capex	51.8	48.2	
33			lifetime	52.6	47.4	

instead important only to minimize immediate costs. This corresponds to the case that we called ”CAPEX optimized” in the previous tests. On the contrary, setting K_{euro} to a high value, implies that big revenue can be earned selling more energy, so it is very important to minimize losses (whatever initial costs this could imply). The balance between the two objectives, in practice, is set by defining the parameter K_{euro} for the specific project of interest. This is a value known by the designer, and varies from project to project. A realistic value for K_{euro} has been used in the tests of the previous subsection (this value considered WACC, subsidies for 10 years of operations and estimated market price). Nevertheless, one could be interested in studying how the balance between immediate costs and long term costs varies when varying K_{euro}. As a practical example, one could be interested in optimizing CAPEX and losses at the same time, but being sure to pay off the extra investment in a short time. We considered, in this test, Horns Rev 3 with cable set cb04. For $K_{euro} = 0$ we have our CAPEX solution of Figure 4 (top), for $K_{euro} = 690$ €/MWh we have our life-time losses optimized solution of Figure 4 (bottom). Company experts estimated 690 €/MWh to be a realistic value for the energy earning over 25 years of operation (expected lifetime of a wind park). We asked them to recompute this value assuming that we want a return of investment in a shorter time. They recomputed it to be $K_{euro} = 176$ for two years, $K_{euro} = 252$ for 3 years, $K_{euro} = 321$ for 4 years, and $K_{euro} = 386$ for 5 years. Setting our balancing factor K_{euro} to these values translates in imposing that extra CAPEX cost will be paid back in 2, 3, 4 or 5 years, respectively. We recomputed the cable costs according to these different values of K_{euro} and re-optimized the layout accordingly. Once the optimized layouts were found, we re-evaluated them with $K_{euro} = 0$ to evaluate their CAPEX costs and $K_{euro} = 690$ to estimate their cost in 25 years. Table 7 shows these figures. For K_{euro} higher than 321 €/MWh the layout is not changing any more. This means that in the lifetime optimized solution ($K_{euro} = 690$) all the additional CAPEX costs were actually paid back in 4 years of operation. In Figure 5 we plot the values from Table 7: the value of the different layouts is decomposed into its CAPEX (x axis) and lifetime-cost part (y axis). The first point (the “+” on the leftmost extreme) represents the value for the CAPEX optimized solution ($K_{euro} = 0$): it has the lowest immediate cost, but the highest cost on the long run. Proceeding from left to right, the next “+”s represent the solutions optimized over 2, 3, 4 and 5 years respectively. From 4 years on, the layout is not changing any more, and equals the solution optimized on the park lifetime ($K_{euro} = 690$), therefore all these layouts are represented at the same coordinates in the plot in Figure 5.

Table 7: Bi-objective analysis for Horns Rev 3 with cable set cb04: changing solutions varying the parameter K_{euro}.

K_{euro}	immediate cost [k€]	total lifetime cost [k€]	revenue loss due to power losses [k€]
0	47283	52663	5379
176	47291	52551	5259
252	47309	52508	5199
321	47325	52490	5165
386	47325	52490	5165
690	47325	52490	5165

5 CONCLUSIONS

In this paper we used a Mixed Integer Linear Programming (MILP) approach to optimize inter-array offshore cable routing considering both the immediate cost of the cables and their power losses during the wind farm lifetime. We proved the importance of using sophisticated optimization tools for this problem. We compared the optimized solution with an existing cable layout, proving that millions of euros can be saved in the given case. We also performed different what-if analyses taking power losses into consideration. Thanks to our optimization methods, we have been able, for the first time, to quantify the impact of considering losses when designing the cable connection of a wind farm. We performed these analyses on

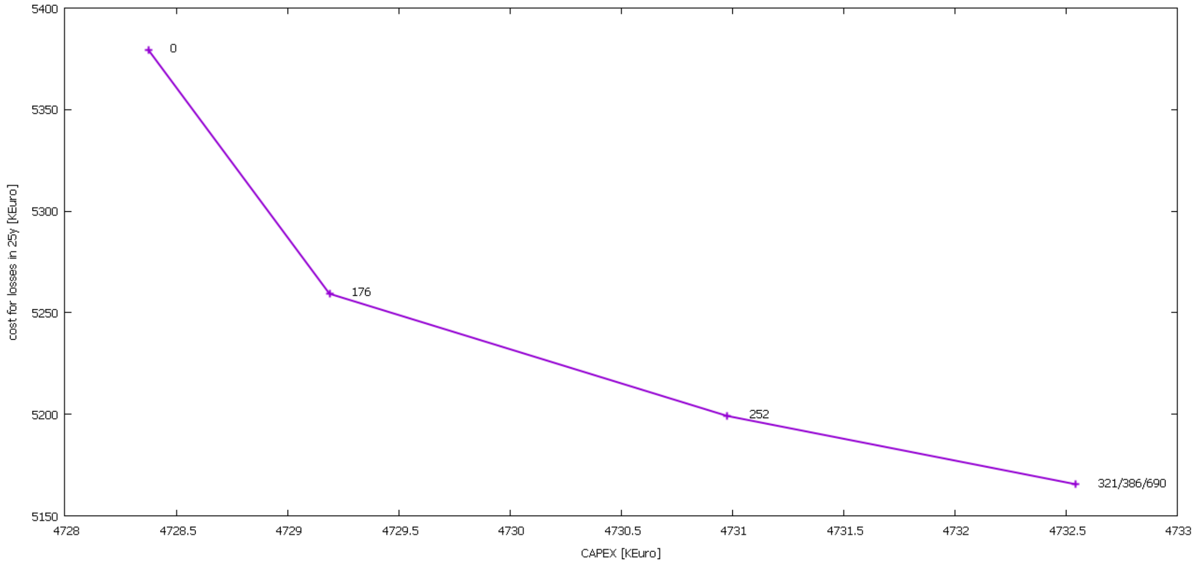

Figure 5: Bi-objective analysis from Table 7. Each "+" corresponds to a layout optimized for a given value of K_{euro} (specified beside each "+") and its coordinates correspond to its immediate cost (x axis) and costs in 25years (y axis). The layouts optimized with $K_{euro} = 321$, 386, and 690 are the same.

a number of real-world instances, analysing the behaviour of the solutions. In general, we observed that it is convenient to invest in cables with less resistance in order to reduce power losses, even if these cables are more expensive at construction time. We used our testbed to evaluate the profitability of the new solutions, both in terms of CAPEX and revenue in the long term. Finally, we performed a Pareto optimality analysis by varying the balancing parameter K_{euro}. This corresponds to giving more or less importance to power losses in the objective function, and is of great importance for designers. In this way, indeed, they can evaluate the return of investment and the impact of their assumptions on the long-term energy price, when designing their cable routing.

ACKNOWLEDGEMENTS

This work was supported by Innovation Fund Denmark. The authors would like to thank Jesper Runge Kristoffersen, Kenneth Skaug, Thomas Hjort and Iulian Vranceanu from Vattenfall BA Wind who helped in defining the cable routing constraints and the cable losses.

REFERENCES

Bauer, J. and Lysgaard, J. (2015). The offshore wind farm array cable layout problem: a planar open vehicle routing problem. *Journal of the Operational Research Society*, (66.3):360–368.

Berzan, C., Veeramachaneni, K., McDermott, J., and Reilly, U. O. (2011). Algorithms for cable network design on large-scale wind farms. Technical report. Massachusetts Institute of Technology.

Cerveira, A., Sousa, A. D., Pires, E. J. S., and Baptista, J. (2016). Optimal cable design of wind farms: The infrastructure and losses cost minimization case. *IEEE Transactions on Power Systems*, (31.6):4319 – 4329.

Dutta, S. (2012). *Data Mining and Graph Theory Focused Solutions to Smart Grid Challenges*. PhD thesis, University of Illinois.

Dutta, S. and Overbye, T. J. (2011). A clustering based wind farm collector system cable layout design. In *Power and Energy Conference at Illinois (PECI)*, pages 1–6.

Fagerfjall, P. (2010). Optimizing wind farm layout: more bang for the buck using mixed integer linear programming.

Fischetti, M. and Pisinger, D. (2016). Optimizing wind farm cable routing considering power losses. Technical report. Danish Technical University.

Gonzlez, J., Payn, M., Santos, J., and Gonzlez-Longatt, F. (2014). A review and recent developments in the optimal wind-turbine micro-siting problem. *Renewable and Sustainable Energy Reviews*, (30):133–144.

Gonzlez-Longatt, F. M. and Wall, P. (2012). Optimal electric network design for a large offshore wind farm based on a modified genetic algorithm approach. *IEEE Systems Journal*, (6.1):164–172.

Hertz, A., Marcotte, O., Mdimagh, A., Carreau, M., and Welt, F. (2012). Optimizing the design of a wind farm collection network. *Information Systems and Operational Research*, (50.2):95–104.

Kristoffersen, J. and Christiansen, P. (2003). Horns rev offshore windfarm: Its main controller and remote control system. *Wind Engineering*, (27.5):351–359.

Li, D., He, C., and Fu, Y. (2008). Optimization of internal electric connection system of large offshore wind farm with hybrid genetic and immune algorithm. In *Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT2008)*, pages 2476–2481.

Pillai, A., Chick, J., Johanning, L., and Laleu, M. K. V. D. (2015). Offshore wind farm electrical cable layout optimization. *Engineering Optimization*, (47.12):1689–1708.

Zhao, M., Chen, Z., and Blaabjerg, F. (2009). Optimisation of electrical system for offshore wind farms via genetic algorithm. *IET Renewable Power Generation 3*, (3.2):205–216.