Fault Estimation using a Takagi-Sugeno Interval Observer: Application to a PEM Fuel Cell

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Abstract: Fault estimation plays an important role in the fault diagnosis system since provides information about the fault magnitude and temporal evolution. In this paper, we present an approach that allows to obtain a simultaneous estimation of the fault, state and associated uncertainty intervals of a uncertain Takagi-Sugeno (TS) system. The fault estimation is obtained using a TS interval observer augmenting the system state with the fault and considering the system uncertainty in a bounded context. A set of Linear Matrix Inequalities (LMIs) have been derived to design the TS interval observer. With the purpose of illustrating the performance of TS interval observer for fault and state estimation, a case study based on a Proton Exchange Membrane (PEM) fuel cell is used.

1 INTRODUCTION

Fault diagnosis involves the fault detection and isolation but also the fault estimation. The fault detection and isolation tasks determine the fault presence in the system (Zhang and Jiang, 2008) (Hwang et al., 2010) (Samy et al., 2011), but not the magnitude. The goal of fault estimation is to provide the size of the fault and its time evolution (Blanke et al., 2006). The fault estimation task is very important for several applications, especially when an active fault-tolerant control (FTC) strategy is implemented (Mahmoud et al., 2003) (Noura et al., 2009) (Witczak, 2014). An example of the application of the fault estimation is to determine the size of the leaks in a pipe system with the aim of quantifying the losses (Brune and F, 2001). There are several approaches for addressing the problem of diagnosis of non-linear systems (Witczak, 2007). In this paper, we can consider that the non-linear model of the system to be monitored can be represented by Takagi-Sugeno (TS) model. TS models were introduced by (Takagi and Sugeno, 1985), and allow describing a nonlinear system as the interpolation of linear models by means of membership functions, that come from a set of fuzzy rules. Different observer design techniques have been developed for TS systems (Guerra et al., 2015) (Aouaouda et al., 2014) (Ichalal et al., 2010) (Zhang et al., 2009).

The presence of uncertainties (unknown parameters, disturbances and/or noise) in the system, complicates the estimation using standard (non-robust) observers. Interval observers can be used to take into account the uncertainty using the set-membership approach (Puig, 2010). The interval observer considers the disturbances, noise and model parameters in a bounded way, evaluating the set of admissible values (interval) for the state vector of each time instant (Efimov et al., 2013b). So far, interval observers have been proposed for the state estimation (Efimov et al., 2013b) and fault detection of nonlinear and LPV systems (Efimov et al., 2013a). As explained in (Rassi et al., 2010), the general idea is to build two observers, which respectively estimate the lower and upper bound of the state vector assuming the system is cooperative. As the original nonlinear system is not cooperative, the observer gain is designed such that the observation error dynamics becomes cooperative. The inclusion of uncertainty in the model parameters in case of an interval observer allows robust fault detection. In (Rotondo et al., 2016), this idea has been used for the fault diagnosis of proton exchange membrane (PEM) fuel cells using a TS interval observer approach. The TS interval observer design is used for the state estimation of TS uncertain systems. Fault diagnosis is addressed using a bank observers where each observer can be made sensitive to different subset of faults.

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The novelty of this paper is to present a fault estimation scheme based on a TS interval observer. This estimator obtain the simultaneous state and fault estimation without making use of the fault detection and isolation modules. Moreover, an interval of the fault magnitude is provided which has not yet been proposed for fault estimation in TS systems. The design of the TS interval observer is addressed with the Lyapunov approach leading to a set of LMIs, that can be efficiently solved using available solvers as YALMIP (Löfberg, 2004). The performance of the TS interval observer for the fault and state estimation is assessed using a case study based on a proton exchange membrane (PEM) fuel cell.

The structure of the paper is the following: In the Section 2, the uncertain TS system is presented. In the Section 3, the formulation of the interval observer design is presented. The proposed PEM fuel cell case study is presented in Section 4, while the results of the application of the proposed approach are presented. Finally, Section 5 presents conclusions of this work.

2 PROBLEM SETUP

2.1 Takagi-Sugeno Uncertain Model

The uncertain TS model of the system to be monitored is expressed in discrete-time including parametric uncertainty as follows:

Rule i : If $\rho_1(k)$ is M_{i1} and $\cdots$ and $\rho_p(k)$ is M_p

$$\text{Then}\begin{cases} x_i(k+1) = (A_i+\Delta A_i)x_i(k)+B_iu(k) \\ \qquad +E_af_a(k)+Dd(k) \\ \\ y_i(k) = Cx_i(k)+E_sf_s(k)+Gv(k) \end{cases} \tag{1}$$

where $i = \{1,2,\cdots,r\}$ and r is the number of rules associated to the different submodels, $x(k) \in \mathbb{R}^{n_x}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the input vector, $y(k) \in \mathbb{R}^{n_y}$ is the measured output vector, $d(k) \in \mathbb{R}^{n_x}$ is the exogenous disturbance, $v(k) \in \mathbb{R}^{n_y}$ is the measurement noise, M denote fuzzy sets and $\rho_1(k),\cdots,\rho_p(k)$ are the premise variables. E_a is fault distribution matrix of actuator faults $f_a(k)$. Analogously, E_s is the fault distribution matrix of sensor faults $f_s(k)$. Combining the local subsystems (1) considering the level of satisfaction of each rule, the following model for the TS system can be obtained:

$$\begin{aligned} x(k+1) = \sum_{i=1}^{r}\xi_i(\rho(k))(A_i+\Delta A_ix(k)+B_iu(k)) \\ +E_af_a(k)+Dd(k) \end{aligned} \tag{2}$$

$$y(k) = Cx(k)+E_sf_s(k)+Gv(k)$$

where $\rho(k) = [\rho_1(k),\cdots,\rho_p(k)]^T$ is the vector containing the premise variables, $\eta_i(\rho(k))$ and $\xi_i(\rho(k))$ are defined as follows:

$$\eta_i(\rho(k)) = \prod_{j=1}^{p} M_{ij}(\rho_j(k)) \tag{3}$$

$$\xi_i(\rho(k)) = \frac{\eta_i(\rho(k))}{\sum_{i=1}^{r}\eta_i(\rho(k))} \tag{4}$$

where $\eta_i(\rho(k))$(product of the membership functions that correspond to the fuzzy sets of a i-th rule) is the membership grade of $\rho_i(k)$ in η_{ij} and $\xi_i(\rho(k))$ is the normalized membership function defined as:

$$\begin{cases} \sum_{i=1}^{r}\xi_i(\rho(k)) = 1 \\ \\ \xi_i(\rho(k)) \geq 0,\, i = \{1,2,\cdots,r\}. \end{cases} \tag{5}$$

The matrices $A_i \in \mathbb{R}^{n_x\times n_x}$, $B_i \in \mathbb{R}^{n_x\times n_u}$ and $C \in \mathbb{R}^{n_y\times n_x}$ contain the system nominal parameters, $\Delta A_i \in \mathbb{R}^{n_x\times n_x}$ represent the parametric uncertainty that is assumed to be not known but bounded $\underline{\Delta A_i} \leq \Delta A_i \leq \overline{\Delta A_i}$. Disturbances $d(k)$ and noise $v(k)$ (assuming that $V(k)$ is the upper bound from measurement noise) are also considered bounded, as follows:

$$\underline{d}(k) \leq d(k) \leq \overline{d}(k) \tag{6}$$

$$|v(k)| \leq V(k) \tag{7}$$

2.2 Fault Estimation Scheme

2.2.1 Problem Formulation

The proposed fault estimation is based on designing a TS interval observer for the uncertain system (2) considering an augmented state vector that considers faults in sensors ($f_s(k)$) and actuators ($f_a(k)$) as follows:

$$\tilde{x}(k) = \begin{bmatrix} x(k) & f_a(k) & f_s(k) \end{bmatrix}^T \tag{8}$$

The TS interval observer will provide an interval estimation of the augmented stated

$$\underline{\hat{\tilde{x}}}(k) \leq \tilde{x}(k) \leq \overline{\hat{\tilde{x}}}(k) \tag{9}$$

i.e., of the states and faults:

$$\underline{\hat{x}}(k) \leq x(k) \leq \overline{\hat{x}}(k) \tag{10}$$

$$\underline{\hat{f}_a}(k) \leq f_a(k) \leq \overline{\hat{f}_a}(k) \tag{11}$$

$$\underline{\hat{f}_s}(k) \leq f_s(k) \leq \overline{\hat{f}_s}(k) \tag{12}$$

When creating the augmented model, it is considered that the actuator fault (11) is modelled as follows:

$$f_a(k+1) = f_a(k) + w_a(k) \tag{13}$$

where $w_a(k) \in \mathbb{R}^f$ considers the actuator fault variations that assumed to be bounded

$$\underline{w}_a(k) \leq w_a(k) \leq \overline{w}_a(k) \tag{14}$$

Similarly, the sensor fault $f_s(k) \in \mathbb{R}^f$ is modelled as follows:

$$f_s(k+1) = f_s(k) + w_s(k) \tag{15}$$

where $w_s(k) \in \mathbb{R}^f$ considers the sensor fault variations that assumed to be bounded

$$\underline{w}_s(k) \leq w_s(k) \leq \overline{w}_s(k) \tag{16}$$

$w_a(k)$ and $w_s(k)$ allowing to consider non-constant faults.

The variations of the faults ($w_a(k)$ and $w_s(k)$) are taken into account altogether with the system disturbance $d(k)$ by means of an augmented disturbance considered for the TS interval observer design as follows:

$$\tilde{d}(k) = \begin{bmatrix} d(k) & w_a(k) & w_s(k) \end{bmatrix}^T \tag{17}$$

Considering the fault models (13) and (15), the augmented model of the system considered for the design the TS interval observer can be expressed in the following form:

$$\begin{aligned} \tilde{x}(k+1) &= \sum_{i=1}^{r} \xi_i(\rho(k))((\tilde{A}_i + \Delta\tilde{A}_i)\tilde{x}(k) + \tilde{B}_i u(k)) \\ &\quad + \tilde{D}\tilde{d}(k) \\ \tilde{y}(k) &= \tilde{C}\tilde{x}(k) + Gv(k) \end{aligned} \tag{18}$$

with:

$$\tilde{A}_i = \begin{bmatrix} A_i & E_a & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}, \tilde{x}(k) = \begin{bmatrix} x(k) \\ f_a(k) \\ f_s(k) \end{bmatrix}$$

$$\Delta\tilde{A}_i = \begin{bmatrix} \Delta A_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & 0 & E_s \end{bmatrix}, \tilde{D} = \begin{bmatrix} D \\ I \\ I \end{bmatrix}$$

where $\tilde{A}_i \in \mathbb{R}^{n_x+n_f \times n_x+n_f}$ are the matrices that contain the matrices of distribution of $f_a(k)$ and $f_s(k)$.

The interval observer for TS system (18), follows the Luenberger form (according to (Rotondo et al., 2016) and (Efimov et al., 2013b)):

$$\begin{aligned} \underline{\hat{\tilde{x}}}(k+1) &= \sum_{i=1}^{r} \xi_i(\rho(k))(\tilde{A}_i - \underline{\tilde{L}}_i\tilde{C})\underline{\hat{\tilde{x}}}(k) + \tilde{B}_i u(k) + (\underline{\Delta\tilde{A}}_i^{+} \\ &\quad \underline{\hat{\tilde{x}}}^{+}(k) - \overline{\Delta\tilde{A}}_i^{+}\overline{\hat{\tilde{x}}}^{-}(k) - \underline{\Delta\tilde{A}}_i^{-}\overline{\hat{\tilde{x}}}^{+}(k) + \overline{\Delta\tilde{A}}_i^{-}\overline{\hat{\tilde{x}}}^{-}(k)) \\ &\quad + \tilde{L}_i y(k) - \left|\underline{\tilde{L}}_i\right| V(k)E_{ny} + \tilde{D}\underline{\tilde{d}}(k) \\ \overline{\hat{\tilde{x}}}(k+1) &= \sum_{i=1}^{r} \xi_i(\rho(k))(\tilde{A}_i - \overline{\tilde{L}}_i\tilde{C})\overline{\hat{\tilde{x}}}(k) + \tilde{B}_i u(k) + (\overline{\Delta\tilde{A}}_i^{+} \\ &\quad \overline{\hat{\tilde{x}}}^{+}(k) - \underline{\Delta\tilde{A}}_i^{+}\overline{\hat{\tilde{x}}}^{-}(k) - \overline{\Delta\tilde{A}}_i^{-}\underline{\hat{\tilde{x}}}^{+}(k) + \underline{\Delta\tilde{A}}_i^{-}\underline{\hat{\tilde{x}}}^{-}(k)) \\ &\quad + \overline{\tilde{L}}_i y(k) + \left|\overline{\tilde{L}}_i\right| V(k)E_{ny} + \tilde{D}\overline{\tilde{d}}(k) \end{aligned} \tag{19}$$

with:

$$\underline{\tilde{L}}_i = \begin{bmatrix} \underline{L}_{i,x} \\ \underline{L}_{i,f_a} \\ \underline{L}_{i,f_s} \end{bmatrix}, \overline{\tilde{L}}_i = \begin{bmatrix} \overline{L}_{i,x} \\ \overline{L}_{i,f_a} \\ \overline{L}_{i,f_s} \end{bmatrix}, \underline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \underline{\hat{x}}_i(k) \\ \underline{\hat{f}}_{i,f_a}(k) \\ \underline{\hat{f}}_{i,f_s}(k) \end{bmatrix}$$

$$\overline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \overline{\hat{x}}_i(k) & \overline{\hat{f}}_{i,f_a}(k) & \overline{\hat{f}}_{i,f_s}(k) \end{bmatrix}^T$$

where $\underline{\tilde{L}}_i \in \mathbb{R}^{n_x+n_f \times n_y}$ and $\overline{\tilde{L}}_i \in \mathbb{R}^{n_x+n_f \times n_y}$ are the observer gains to be designed, $\overline{\Delta\tilde{A}}_i^{+} = \max\left\{0, \overline{\Delta\tilde{A}}_i\right\}$, $\overline{\Delta\tilde{A}}_i^{-} = \overline{\Delta\tilde{A}}_i^{+} - \overline{\Delta\tilde{A}}_i$, $\underline{\Delta\tilde{A}}_i^{+} = \max\left\{0, \underline{\Delta\tilde{A}}_i\right\}$, $\underline{\Delta\tilde{A}}_i^{-} = \underline{\Delta\tilde{A}}_i^{+} - \underline{\Delta\tilde{A}}_i$, $\overline{\hat{\tilde{x}}}^{+} = \max\left\{0, \overline{\hat{\tilde{x}}}\right\}$, $\overline{\hat{\tilde{x}}}^{-} = \overline{\hat{\tilde{x}}}^{+} - \overline{\hat{\tilde{x}}}$, $\underline{\hat{\tilde{x}}}^{+} = \max\left\{0, \underline{\hat{\tilde{x}}}\right\}$, $\underline{\hat{\tilde{x}}}^{-} = \underline{\hat{\tilde{x}}}^{+} - \underline{\hat{\tilde{x}}}$ and, finally $E_{n_y} \in \mathbb{R}^{n_y \times 1}$ is the column vector with elements equal to 1. $\overline{\tilde{d}}(k)$ and $\underline{\tilde{d}}(k)$ are the bounds from (17). The output interval estimation of $y(k)$ can be obtained as follows: $\underline{\hat{\tilde{y}}} = \tilde{C}\,\underline{\hat{\tilde{x}}}$ and $\overline{\hat{\tilde{y}}} = \tilde{C}\,\overline{\hat{\tilde{x}}}$.

To design the TS interval observer for the system (2) with the augmented state vector including the faults (13) and (15) should be observable. Observability of the TS uncertain system (2) can be assessed using the approach proposed in (Ho et al., 2013). If the following, it is assumed that this observability condition for the augmented system (18) is satisfied.

2.2.2 Integration with FDI

The proposed fault estimation scheme could be integrated with a FDI scheme as the one proposed in (Rotondo et al., 2016) and only activated once the fault has been detected and isolated. In particular, a bank of a bank of n_f dedicated observers where each observer has been designed to estimate only one fault

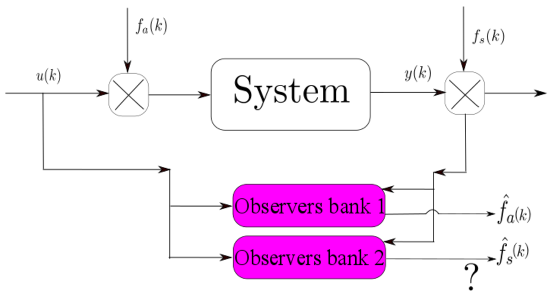

Figure 1: Integration with FDI using a bank of observers.

will be designed. Then, the TS interval observer that considers as additional state the fault that has been detected and isolated will be used for estimating the fault. Alternatively, the fault estimation scheme can be integrated with a bank of n_f dedicated observers where each observer has been designed to be sensitive to only one fault (using as e.g., an unknown input observer approach (Blanke et al., 2006)), as shown in Fig. 1. In this second case, the following fault detection and isolation logic can be used: While the fault estimation intervals provided by all the observers in the bank satisfy

$$0 \in [\underline{\hat{f}}_i, \overline{\hat{f}}_i] \quad i = 1, \cdots, n_f$$

no fault is detected. Otherwise, when some of the observers provides a fault estimation interval satisfying

$$0 \notin [\underline{\hat{f}}_i, \overline{\hat{f}}_i]$$

it means that the fault f_i has been ocurred, being the fault magnitude bounded by $[\underline{\hat{f}}_i, \overline{\hat{f}}_i]$

3 TAKAGI-SUGENO INTERVAL OBSERVER DESIGN

To design a TS interval observer of the form (19) that ensures (9), (and therefore (10), (11) and (12)) with acceptable performance specified by means of an LMI region, the following theorem is introduced.

Theorem 1. *Given positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, a TS interval observer* (19) *with performance defined with an LMI region defined by two vertical strips (h_1 and h_2) and a disk (r and q are the radius and center) can be obtained, if there exist matrices $P = P^T > 0$, $Q > 0$ and $\underline{\widetilde{W}}_i \in \mathbb{R}^{2n_x + n_f \times 2n_u}$ $\overline{\widetilde{W}}_i \in \mathbb{R}^{2n_x + n_f \times 2n_u}$ that satisfy the following inequalities for $i = \{1, 2, \cdots, r\}$:*

$$\begin{bmatrix} \frac{P}{1+\varepsilon_1} & PD_{ai} - W_i\Upsilon & \frac{P}{1+\varepsilon_1} \\ (PD_{ai} - W_i\Upsilon)^T & P - Q - \lambda\eta^2 I_{2n_x} & 0 \\ \frac{P}{1+\varepsilon_1} & 0 & \lambda I_{2n_x} - \tau P \end{bmatrix} \geq 0 \tag{20}$$

$$P \begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix} - W_i\Upsilon \geq 0 \tag{21}$$

$$\begin{bmatrix} -rP & * \\ qP + P \begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix} - W_i\Upsilon & -rP \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix} P + P \begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix}^T + 2h_2 P < 0 \tag{23}$$

$$\begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix} P + P \begin{bmatrix} \widetilde{A}_i & 0 \\ 0 & \widetilde{A}_i \end{bmatrix}^T + 2h_1 P > 0 \tag{24}$$

with $W_i = \begin{bmatrix} \underline{\widetilde{W}}_i & 0 \\ 0 & \overline{\widetilde{W}}_i \end{bmatrix}$, $\lambda > 0$, $\tau = 1 + \varepsilon_2 + (1 + \varepsilon_1)^{-1}$

$$D_{ai} = \begin{bmatrix} \widetilde{A}_i + \underline{\widetilde{\Delta A}}_i^{+} & 0 \\ 0 & \widetilde{A}_i + \overline{\widetilde{\Delta A}}_i^{+} \end{bmatrix}$$

$$\Upsilon = \begin{bmatrix} \widetilde{C} & 0 \\ 0 & \widetilde{C} \end{bmatrix}$$

Proof: The theorem can be easily proved adapting the results of (Efimov et al., 2013b) for LPV systems and (Chilali and Gahinet, 1996) for pole placement in one the LMI region.

Then, the gains of the interval observer (19) are obtained after solving the inequalities (20), (21), (22), (23) and (24)) as follows

$$\underline{\widetilde{L}}_i = P^{-1}\underline{\widetilde{W}}_i \text{ and } \overline{\widetilde{L}}_i = P^{-1}\overline{\widetilde{W}}_i \tag{25}$$

with $i = \{1, 2, \cdots, r\}$.

4 CASE STUDY

4.1 Fuel Cell System

Fuel cells have been considered as alternative energy sources for the future with potential application to several areas: transport (buses, trucks, trains, etc), military applications (portable soldier power), auxiliary power units, and electricity generation provide electricity (and sometimes heat) (Wee, 2007). Proton exchange membrane fuel cells (PEMFC) are electromechanical devices in which the energy of a reaction between a fuel, the hydrogen, and oxidant, the oxygen, is directly and continuously converted into electrical energy, obtaining water as a subproduct (Pukrushpan et al., 2004). In the literature, we can find works dealing with PEMFC, considering durability, optimal control, model predictive control, fault diagnosis and different approaches for fuell cell modeling (electric equivalent model, state space model,etc) (Rotondo et al., 2016).

The nonlinear model of the PEMFC (Pukrushpan et al., 2004), can be derivate by four subsystems: compressor, supply manifold, cathode plus return manifold and anode. For illustrating purposes and because of space limitations, the scheme proposed in this paper only the compressor subsystem is used.

4.2 Compressor System

The model of the compressor is described by means of the following equation:

$$\dot{\omega}_{cp} = \left(-\frac{\tilde{Z}_3 V_{cp}}{J_{cp} R_{cm}} K_v\right)\omega_{cp} + \frac{\tilde{Z}_3}{J_{cp}}\left(\frac{V_{cp}}{R_{cm}} - \frac{C_p T_{amb}}{\eta_{cm}\tilde{\eta}_{cp}}\tilde{Z}_1\tilde{Z}_2\right) \tag{26}$$

where ω_{cp} is the compressor speed, K_v is the motor electric constant, J_{cp} is the compressor and motor inertia, R_{cm} is the compressor motor circuit resistance, C_p is the air heat capacity at constant pressure, T_{amb} is the ambient temperature and η_{cm} is the compressor efficiency V_{cp} is the voltage, $\tilde{Z}_{1,2,3}$ are function of the stack current I_{st}. For assessing the nonlinear compressor model the values and parameters of the Table 1 and 2 are considered.

The model (26) can be rewritten of the form (2) with the nonlinear sector approach, considering the following that $\rho(k) = I_{st}(k)$ and the state space matrices are:

$$A(\rho(k)) = \left(-\frac{\tilde{Z}_3 V_{cp}}{J_{cp} R_{cm}} K_v\right)$$

$$B(\rho(k)) = \frac{\tilde{Z}_3}{J_{cp}}\left(\frac{V_{cp}}{R_{cm}} - \frac{C_p T_{amb}}{\eta_{cm}\tilde{\eta}_{cp}}\tilde{Z}_1\tilde{Z}_2\right)$$

where ω_{cp} is the state, V_{cp} is the input, I_{st} is the variable parameter. The dimensions of the matrices are $A(\rho(k)) \in \mathbb{R}^{1\times 1}$ and $B(\rho(k)) \in \mathbb{R}^{1\times 1}$.

The vertices models are scheduled under the following:

$\overline{\rho}(k) = \frac{I_{st}(k) - I_{stmin}}{I_{stmax} - I_{stmin}}$ and $\underline{\rho}(k) = \frac{I_{stmax} - I_{st}(k)}{I_{stmax} - I_{stmin}}$.

Then, in order to obtain the results of the scheme proposed is considered that the stack current is in the range $I_{st} \in [100, 300]\, mA$. Then, using a sequence of steps of $10\ mA$ a set of 21 possible operating points are obtained. The systems is discretized with a sampling time $T_s = 0.01\, s$.

Table 1: Parameters of the nonlinear model of a compressor.

Symbol	Value
k_v	$0.0153\ V/(rad/s)$
J_{cp}	$5\times 10^{-5}\ kg\cdot m^2$
R_{cm}	$0.816\ \Omega$
C_p	$1004\ J/(kg\cdot K)$
T_{amb}	$298\ K$
η_{cm}	0.9

Table 2: Variables in the nonlinear model of a compressor.

Approximate variables
$\tilde{\eta}_{cp} \approx 0.777217$
$\tilde{Z}_1 \approx 0.275641 \cdot 10^{-3} I_{st} - 0.340993 \cdot 10^{-3}$
$\tilde{Z}_2 \approx 0.001375 I_{st} - 0.023710$
$\tilde{Z}_3 \approx -0.000426 \cdot 10^{-3} I_{st} + 0.213459 \cdot 10^{-3}$

4.3 Actuator Fault Estimation

For the actuator fault estimation, the following dedicated observer is used:

$$\begin{cases} \underline{\hat{\tilde{x}}}(k+1) = & \sum_{i=1}^{r}\xi_i(\rho(k))(\tilde{A}_i - \underline{\tilde{L}}_i[C\ 0])\underline{\hat{\tilde{x}}}(k) + \tilde{B}_i u(k) \\ & +\underline{\widetilde{\Delta A}}_i^{+}\underline{\hat{\tilde{x}}}^{+}(k) - \overline{\widetilde{\Delta A}}_i^{+}\overline{\hat{\tilde{x}}}^{-}(k) - \underline{\widetilde{\Delta A}}^{-}\overline{\hat{\tilde{x}}}^{+}(k) \\ & +\overline{\widetilde{\Delta A}}_i^{-}\overline{\hat{\tilde{x}}}^{-}(k) + \underline{\tilde{L}}_i y(k) - \left|\underline{\tilde{L}}_i\right| V(k) E_{ny} + \tilde{D}\underline{\tilde{d}}(k) \\ \overline{\hat{\tilde{x}}}(k+1) = & \sum_{i=1}^{r}\xi_i(\rho(k))(\tilde{A}_i - \overline{\tilde{L}}_i[C\ 0])\overline{\hat{\tilde{x}}}(k) + \tilde{B}_i u(k) \\ & +\overline{\widetilde{\Delta A}}_i^{+}\overline{\hat{\tilde{x}}}^{+}(k) - \underline{\widetilde{\Delta A}}_i^{+}\overline{\hat{\tilde{x}}}^{-}(k) - \overline{\widetilde{\Delta A}}_i^{-}\underline{\hat{\tilde{x}}}^{+}(k) + \underline{\widetilde{\Delta A}}_i^{-} \\ & \underline{\hat{\tilde{x}}}^{-}(k) + \overline{\tilde{L}}_i y(k) + \left|\overline{\tilde{L}}_i\right| V(k) E_{ny} + \tilde{D}\overline{\tilde{d}}(k) \end{cases} \tag{27}$$

with:

$$\tilde{A}_i = \begin{bmatrix} A_i & E_a \\ 0 & I \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \underline{\tilde{L}}_i = \begin{bmatrix} \underline{L}_{i,x} \\ \underline{L}_{i,f_a} \end{bmatrix}$$

$$\overline{\tilde{L}}_i = \begin{bmatrix} \overline{L}_{i,x} \\ \overline{L}_{i,f_a} \end{bmatrix}, \underline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \underline{\hat{x}}(k) \\ \underline{\hat{f}}_{i,f_a}(k) \end{bmatrix}, \overline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \overline{\hat{x}}(k) \\ \overline{\hat{f}}_{i,f_a}(k) \end{bmatrix}$$

$$\underline{\widetilde{\Delta A}}_i^{+} = \begin{bmatrix} \underline{\Delta A}_i^{+} & 0 \\ 0 & 0 \end{bmatrix}, \overline{\widetilde{\Delta A}}_i^{+} = \begin{bmatrix} \overline{\Delta A}_i^{+} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\overline{\widetilde{\Delta A}}_i^{-} = \begin{bmatrix} \overline{\Delta A}_i^{-} & 0 \\ 0 & 0 \end{bmatrix}, \underline{\widetilde{\Delta A}}_i^{-} = \begin{bmatrix} \underline{\Delta A}_i^{-} & 0 \\ 0 & 0 \end{bmatrix}, \tilde{D} = \begin{bmatrix} D \\ I \end{bmatrix}$$

The observer gains are placed in a LMI region, defined as the intersection of a disk sector with $r = 1$, $q = 0$ and two vertical strips: $h_1 = -0.11$ and $h_2 = -0.9$.

To assess the performance of fault estimation provided by the TS interval observer, two type of faults are considered (abrupt fault and incipient fault).

Case 1.- Abrupt Fault

With the aim of showing the estimator performance in case of an actuator fault, an abrupt change of 8 to 22 sec is made. Figure 2 shows that although there exists a fault, the observer is able to estimate ω_{cp}.

Figure 3 presents the fault estimation (f_a) where the estimator is slow when the fault occurs but after a transient it can provide an interval for the fault estimation.

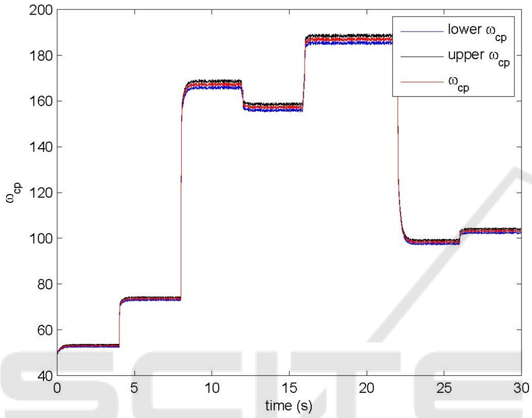

Figure 2: Estimation of ω_{cp}.

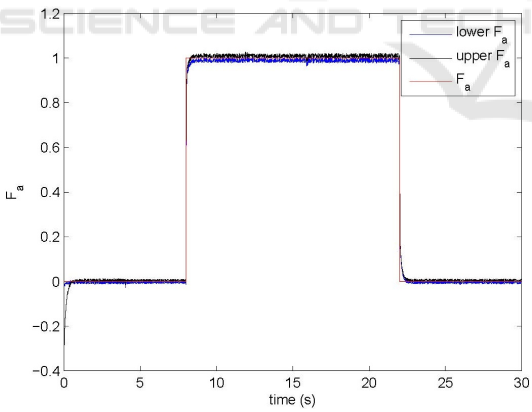

Figure 3: Estimation of f_a.

Case 2.- Incipient Fault

Considering an incipient fault scenario, we can observe that the observer is able to estimate this type of fault, Figure 4 shows the state estimation ω_{cp} while Figure 5 illustrates the fault estimation. From these two figures, it can be seen that the observer (27) is able to estimate adequately the fault and the state.

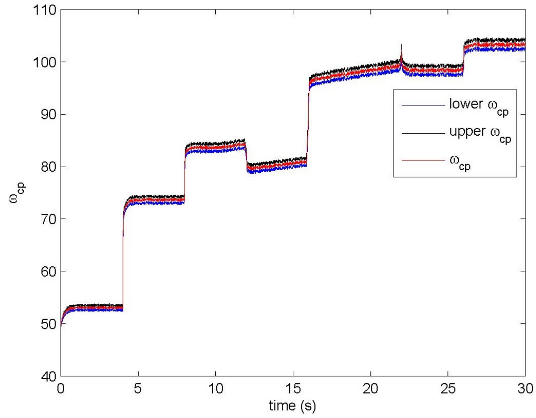

Figure 4: Estimation of ω_{cp}.

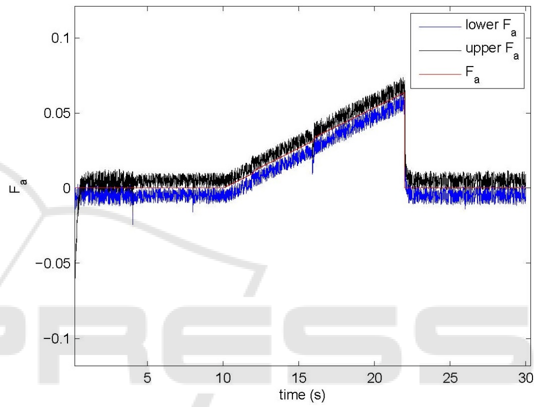

Figure 5: Estimation of f_a.

4.4 Sensor Fault Estimation

For the sensor fault estimation, the following TS interval observer is used:

$$\begin{cases} \underline{\hat{\tilde{x}}}(k+1) = & \sum_{i=1}^{r} \xi_i(\rho(k))(\widetilde{A}_i - \underline{\widetilde{L}}_i[C\ E_s])\underline{\hat{\tilde{x}}}(k) + \widetilde{B}_i u(k) \\ & + \underline{\widetilde{\Delta A}}_i^{+} \underline{\hat{\tilde{x}}}^{+}(k) - \overline{\widetilde{\Delta A}}_i^{+} \overline{\hat{\tilde{x}}}^{-}(k) - \underline{\widetilde{\Delta A}}^{-} \overline{\hat{\tilde{x}}}^{+}(k) + \overline{\widetilde{\Delta A}}_i^{-} \\ & \overline{\hat{\tilde{x}}}^{-}(k) + \underline{\widetilde{L}}_i y(k) - \left|\underline{\widetilde{L}}_i\right| V(k) E_{ny} + \widetilde{D}\underline{\widetilde{d}}(k) \\ \overline{\hat{\tilde{x}}}(k+1) = & \sum_{i=1}^{r} \xi_i(\rho(k))(\widetilde{A}_i - \overline{\widetilde{L}}_i[C\ E_s])\overline{\hat{\tilde{x}}}(k) + \widetilde{B}_i u(k) \\ & + \overline{\widetilde{\Delta A}}_i^{+} \overline{\hat{\tilde{x}}}^{+}(k) - \underline{\widetilde{\Delta A}}_i^{+} \overline{\hat{\tilde{x}}}^{-}(k) - \overline{\widetilde{\Delta A}}_i^{-} \underline{\hat{\tilde{x}}}^{+}(k) + \underline{\widetilde{\Delta A}}_i^{-} \\ & \underline{\hat{x}}^{-}(k) + \overline{\widetilde{L}}_i y(k) + \left|\overline{\widetilde{L}}_i\right| V(k) E_{ny} + \widetilde{D}\overline{\widetilde{d}}(k) \end{cases} \quad (28)$$

with:

$$\widetilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & I \end{bmatrix}, \widetilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \underline{\widetilde{L}}_i = \begin{bmatrix} \underline{L}_{i,x} \\ \underline{L}_{i,f_s} \end{bmatrix}$$

$$\overline{\widetilde{L}}_i = \begin{bmatrix} \overline{L}_{i,x} \\ \overline{L}_{i,f_s} \end{bmatrix}, \underline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \underline{\hat{x}}(k) \\ \underline{\hat{f}}_{i,f_s}(k) \end{bmatrix}, \overline{\hat{\tilde{x}}}(k) = \begin{bmatrix} \overline{\hat{x}}(k) \\ \overline{\hat{f}}_{i,f_s}(k) \end{bmatrix}$$

$$\widetilde{\underline{\Delta A}}_i^+ = \begin{bmatrix} \underline{\Delta A}_i^+ & 0 \\ 0 & 0 \end{bmatrix}, \widetilde{\overline{\Delta A}}_i^+ = \begin{bmatrix} \overline{\Delta A}_i^+ & 0 \\ 0 & 0 \end{bmatrix}$$

$$\widetilde{\overline{\Delta A}}_i^- = \begin{bmatrix} \overline{\Delta A}_i^- & 0 \\ 0 & 0 \end{bmatrix}, \widetilde{\underline{\Delta A}}_i^- = \begin{bmatrix} \underline{\Delta A}_i^- & 0 \\ 0 & 0 \end{bmatrix}, \widetilde{D} = \begin{bmatrix} D \\ I \end{bmatrix}$$

In this case, the LMI region is defined by a disk sector with $r = 1$, $q = 0$, $h_1 = -0.81$ and $h_2 = -0.99$ are considered for the vertical strips.

Case 1.- Abrupt Fault.

An abrupt fault occurs from 18 to 25 sec affecting the speed sensor. Figure 6 presents the interval estimation that for this case the observer (28) can estimate.

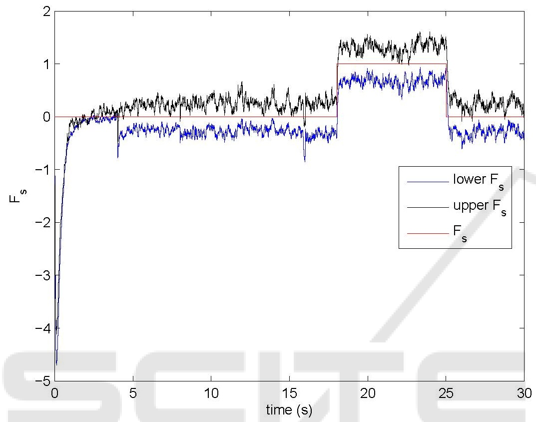

Figure 6: Estimation of f_s.

The state estimation of the TS system with the fault affecting the speed sensor is shown in the Figure 7.

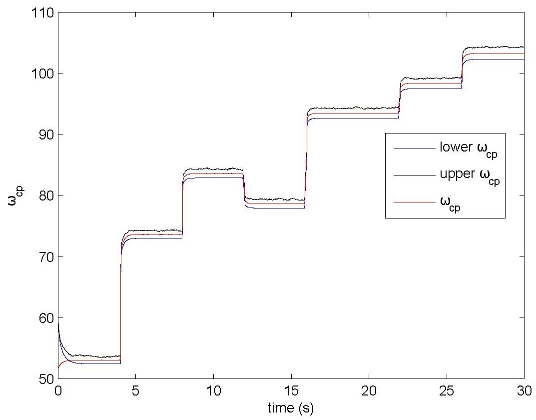

Figure 7: Estimation of ω_{cp}.

Case 2.- Incipient Fault.

Figure 8 illustrates the interval fault estimation when the incipient fault is applied in the output sensor. The state estimation is presented in the Figure 9.

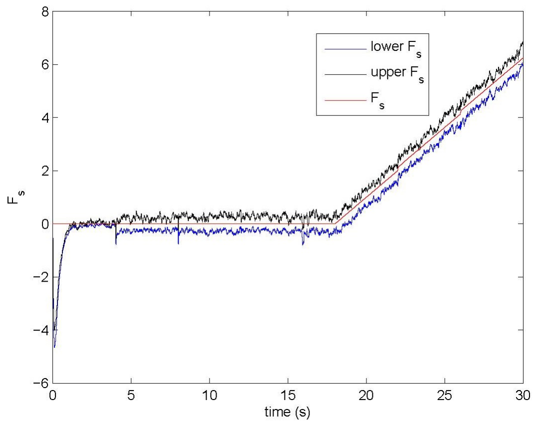

Figure 8: Estimation of f_s.

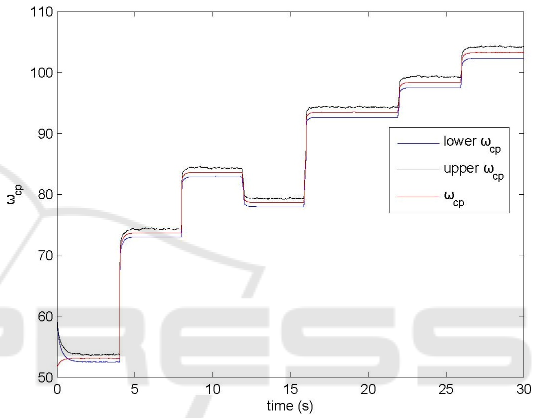

Figure 9: Estimation of ω_{cp}.

5 CONCLUSION

In this paper, an approach for estimating faults using the TS interval observer is proposed. Faults are considered as an additional state in the observer model. The proposed method allows to obtain the estimation of faults and state simultaneous. In case that observability conditions are satisfied FDI is not required. Otherwise, it should be used combined with a bank of observers dedicated to a subset of faults. The proposed approach have been satisfactorily tested on the compressor of fuel cell system in several scenarios including abrupt and incipient faults. As a future work, the proposed scheme will extended to process faults and to the case of unmeasured premise variables.

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