Excellent Potential of Geometric Brownian Motion (GBM) as a Random Process Model for Level of Drowsiness Signals

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Abstract: We show that Geometric Brownian Motion (GBM) appears to be an excellent choice of random process model to describe mathematically the real-life signals that represent the evolution with time of the level of drowsiness (LoD) of an individual, such as a driver. We collected data from thirty (30) healthy participants, who each underwent three tests (either driving in a simulator or performing Psychomotor Vigilance Tests) at successive levels of sleep deprivation. During each test, the LoD was produced by a photooculography (POG) based device designed and built by our team. We so obtained a total of 90 LoD signals. For each, we applied statistical methods to determine whether a GBM was a valid model for it. All 90 signals passed statistical tests of normality and independency, meaning that each can be modeled by GBM, thereby showing the excellent potential of GBM as a random process model for LoD signals. This finding could lead to the development of a number of innovative means for predicting the evolution of the LoD and the occurrence of related events beyond the present moment. The resulting technology should help reduce the number of accidents due to drowsy driving.

1 INTRODUCTION

The drowsy state is an intermediate state between alert wakefulness and sleep as defined electrophysiologically by the pattern of brain waves (EEG), eye movements (EOG), and muscle activity (EMG) (Johns, 2001, p. 5). Drowsiness is a major cause of accidents in many areas of human activity (whether personal or professional), and transportation is probably the single most important source of drowsiness-related accidents. For example, one third (1/3) of fatal accidents on highways in France are reported due to the driver falling asleep at the wheel (Association des Sociétés Françaises d'Autoroutes, 2010).

It is thus paramount to monitor the level of drowsiness (LoD) of a driver and to devise in-car safety systems that can help prevent accidents due to drowsiness. This implies the development and use of drowsiness monitoring systems. We focus here on systems that monitor the physiological state of the subject, e.g. by using images of an eye.

All systems that we know of can establish a present LoD based on such data obtained up to the present time. But, if the LoD at the present time reaches a critical level, it may be too late to save a driver's life. Therefore, there is an imperative need to estimate how the LoD will evolve past the present time, and to make LoD-related predictions. To the best of our knowledge, the field of predicting the evolution of drowsiness and of related parameters in the future is essentially completely unexplored. The first investigations of this question appear to be (Ebrahimbabaie and Verly, 2016a; Ebrahimbabaie and Verly, 2016b; Ebrahimbabaie and Verly, 2016c).

A conventional strategy for predicting future values of a signal is to describe this signal via a model. Since the evolution of the LoD is inherently random, one must treat each real-life "LoD signal" as a realization of a random process (RP).

The RP process models that often come to mind in a wide variety of applications are AR, ARMA, and ARIMA models, where "AR" stands for "autoregressive", "MA" for "moving average", and "I" for "integrated" (Manolakis et al., 2005; Hayes, 1996). In (Ebrahimbabaie and Verly, 2016a; Ebrahimbabaie and Verly, 2016b; Ebrahimbabaie and Verly, 2016c), we examined the application of such models to a PERCLOS signal and to an LoD signal, and we found that these signals could be properly modeled by AR(I)MA RP models. However, we also pointed

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out that these models are quite heavy to deal with, in part because, for each, the order is unknown and the parameters are unknown and numerous.

Our search for better RP models in the context of drowsiness monitoring led us to examine the Geometric Brownian Motion (GBM) RP model (Jeanblanc et al., 2009). A preliminary investigation indicated that the GBM RP model could be very appropriate for the signals found in the context of drowsiness monitoring, such as a PERCLOS signal and an LoD signal.

The main goal of the present paper is to describe the work that we did with real subjects in several states of sleep deprivation to establish that the GBM RP model appears to be a good, promising choice of RP model to describe the LoD signals produced by a specific, validated, POG-based drowsiness quantification instrument, at least based on the data we had.

The GBM RP model lies at the heart of this paper. We now give a brief definition of what a GBM RP is. As one shall see, the notion of a GBM RP is rooted in significantly advanced mathematics (Jeanblanc et al., 2009).

A continuous-time RP X(t) is said to be a GBM, or GBM - i.e. to follow a GBM (RP) model - if it satisfies the stochastic differential equation (SDE)

$$\frac{dX(t)}{X(t)} = \mu dt + \sigma dW(t), \qquad (1)$$

where μ is a fixed, real-valued parameter, σ a fixed, real, positive parameter, and $W(t)$ a Weiner (random) process also called Brownian Motion (BM) (Jeanblanc et al., 2009).

The left side of Equation 1 is the relative increment of $X(t)$ in the period of time $[t, t+dt]$, i.e. $(X(t+dt) - X(t))/X(t)$. The right side of this equation shows that this relative increment has a deterministic linear trend μdt that is disturbed by a random noise term $\sigma dW(t)$. The constant μ is the so-called "drift" (or "mean rate of return" in financial mathematics), and σ is the so-called "volatility".

Recall that the goal of this paper is to show that GBM is a good RP model for real-life LoD signals.

2 METHODS

We used data from two laboratory-based studies, referred to here as Study A and Study B. Both studies used the same overall (experimental) protocol, and they differed only by (1) the groups of participants/subjects who took part in each study, and (2) the nature of the tests/tasks that each participant was asked to submit to in each study.

2.1 Participants

We recorded experimental data from N = 30 healthy participants aged 19-33. Study A contributed 13 subjects (mean age: 23.7; 7 men, 6 women), and Study B 17 subjects (mean age: 22.7; 8 men, 9 women).

2.2 Protocol

In each of the two studies (A and B), the corresponding participants were each asked to submit to three successive, time-separated test sessions in different sleep-deprivation conditions over two days. During each test session, the LoD signal of each participant was produced using a drowsiness monitoring system designed, built, and validated by our team.

In Study A, each test session consisted in driving in a high-fidelity driving simulator; the three successive sessions had durations of 45, 45, and 60 minutes. In Study B, each test session consisted in performing a Psychomotor Vigilance Test (PVT); the three successive sessions all had durations of 10 minutes.

For ease of explanation, the overall two-day experiment for each participant (for either type of test) can be viewed as the succession of Night 1, Day 1, Night 2, and Day 2, and as consisting of three successive test sessions. Figure 1 provides an illustration of the overall protocol used for both studies. On Night 1, the participant slept at home and was asked to report the number of hours of sleep using a sleep diary (mean $\pm$ standard deviation for all participants is 7.57 $\pm$ 0.8 h of sleep, range 6.5–9.0 h). Then, the participant was not allowed to sleep from the time he/she woke up on Day 1 until the end of the study (12:00 noon on Day 2). (All times are in 24 h notation.) At 8:00 on Day 1, the participant arrived at our laboratory and submitted to the first test session, between 8:00 and 10:00. The participant was then free to leave the laboratory to carry out his/her normal activities but was equipped with an actigraph (either Actiwatch 2 or Philips Respironics) in order to check that he/she had not slept while away. The participant came back to our laboratory at 20:30 on Day 1. On Night 2, the participant submitted to the second test session between 2:00 and 4:00 and, after breakfast on Day 2, he/she submitted to the third test session between 11:00 and 13:00 (and after at least 28 hours of sleep deprivation). At the end of the study, the participant was sent back home. From noon on Day 1 until the end of third test session, the participant was asked not to consume any stimulant (coffee, tea, etc.). This protocol was approved by the Ethics Committee of the University of Liège (François et al., 2016).

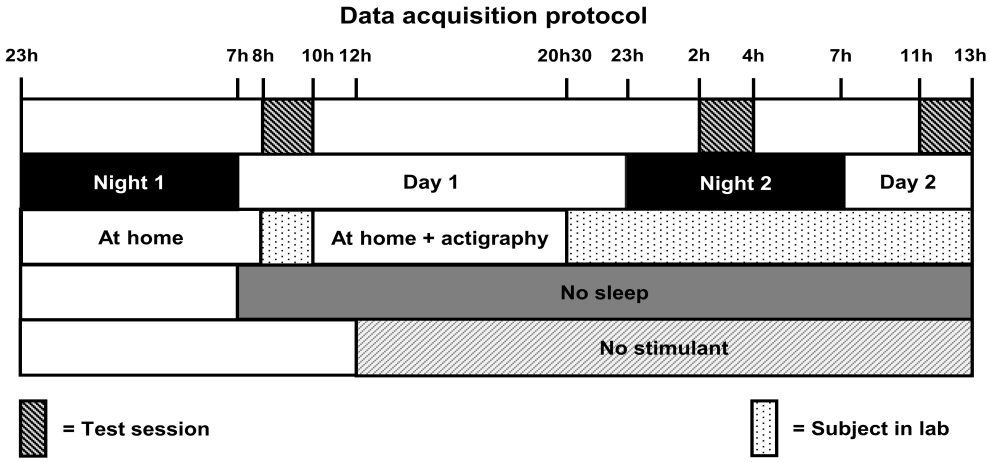

Figure 1: Graphical overview of overall protocol common to both studies, i.e. for Study A (driving in a simulator) and Study B (PVT). The successive five lines show: (1) when each participant had to submit to a test; (2) the succession of nights and days; (3) the presence at home or in our lab; (4) the period of sleep deprivation; (5) the period of stimulant deprivation.

2.3 Measurements

We setup our drowsiness monitoring system to produce one sample of the LoD every 20 sec in Study A and every 5 sec in Study B. One reason for considering two distinct sampling intervals was to examine whether the conclusion concerning the nature of the underlying RP for LoD signals would dependent on the value of the sampling interval. One other reason for decreasing the sampling interval to 5 sec - thereby producing LoD signals with 4 times as many samples - was to provide more data for the statistical analysis.

2.4 Instruments

Our team designed, built, and validated (François et al., 2016; François et al., 2014) a fully automatic drowsiness monitoring system based on the physiological state of a person. The system consists in a pair of specially-made eyeglasses linked to a computer via a cable. The eyeglasses contain a controlled source of illumination in the infrared (IR) and a high-speed camera sensitive in the IR.

The algorithms running on the computer use ocular parameters extracted from images of the eye (i.e. POG) to determine, periodically in time, an LoD on a numerical scale from 0 to 10, with 0 corresponding to ”very awake” (or ”very vigilant”) and 10 to ”very drowsy” (François et al., 2016).

2.5 Data Analysis

The sequence of LoD values produced by our drowsiness monitoring system during each particular test session is referred to here as a signal. The signals recorded during Study A and Study B are further described in the next section. In an actual operational situation, the signal would be the sequence of samples produced during the time the operator has the glasses on, such as during a long drive.

Consider a given, real-life signal - such as the one just described - that is considered to be one realization of a RP. Here, we describe the methodology that is typically used to determine whether the GBM RP model is a valid model for this real-life signal.

Before describing this methodology, we introduce some key concepts. For ease of explanation, it is useful to treat a RP as a sequence of random variables (RVs), thus ordered in time. However, rather than considering these RVs, one works here with the related log-ratio (LR) RVs. The LR RV at some time index n is defined as the natural logarithm of the ratio of the RV at n and the RV at n - 1. The collection of LR RVs constitutes a new RP, called here the LR RP. (Note that the number of RVs in the LR RP is necessarily one unit less that in the original RP.) Of course, in practice, we (only) have one realization of the original RP, and thus one realization of the LR RP, and, consequently, one realization of each original RV or LR RV. While all theoretical developments must be done in terms of RPs and RVs, the methodology only uses the single available realization of the original RP and that of the corresponding LR RP, as well as of their corresponding RVs.

At the theoretical level, if the original RP is a GBM RP, then one can show that the LR RP is characterized by the following pair of properties: (1) all LR RVs are individually characterized by a normal (i.e. Gaussian) PDF, and the PDFs of all these RVs are identical (thus with the same mean and standard deviation); (2) any two distinct LR RVs are indepen-

dent in the usual statistical sense, meaning that their joint (2D) PDF factorizes (Brandimarte, 2014).

At the methodology level, one must determine whether both of the above properties (or conditions) are verified. The difficulty - which is typical in the type of approach used here - is that the theory deals with all possible realizations, while the methodology has only access to a single one, i.e. the observed signal. The methodology must thus do its best with a single signal/realization to establish one or more properties that pertain to all possible realizations. In short, the methodology must replace considerations across a statistical ensemble by considerations across time. Below, we simply describe the methodology, and we do not attempt to justify it in terms of the theory.

In light of the above discussion, there are thus two conditions to be verified on the signal/realization available to establish whether this realization can be modelled by a GBM RP. We refer to these conditions as the **"normality condition"** and the **"independency condition"**.

In order to verify normality condition, we applied, to each signal, (1) two established graphical methods, i.e. the quantile-quantile (QQ) plot and the histogram, and (2) one numerical (non-graphical) method, i.e. the Shapiro-Wilk (S-W) test.

To assess the independency condition, we looked at the scatter plot of Log-Ratios versus time of each signal to see whether there was any (time) correlation between the logarithms of the ratios of successive values.

3 RESULTS

3.1 Data

Each participant in Study A and Study B contributed 3 LoD signals, with each signal corresponding to one of the 3 tests at progressively increasing levels of sleep deprivation. Thus, the 13 participants of Study A (driving in a simulator) contributed 13 x 3 = 39 LoD signals, and the 17 participants of Study B (performing PVTs) contributed 17 x 3 = 51 LoD signals.

As alluded to earlier, some characteristics of the LoD signals produced are different for each study. In Study A, each signal consists of 42 samples spaced by 20 seconds (for a total duration of 840 sec = 14 min), and in Study B, each signal consists of 110 samples spaced by 5 seconds (for a total duration of 550 sec = 9 min 10 sec). The time intervals of 20 and 5 sec are those at which the drowsiness measurement system was asked to produce its output samples. This should be distinguished from the speed at which the images of the eye were collected, i.e. 120 images/sec in both Study A and Study B.

In summary, we have the following numbers of LoD signals and samples:

- Study A: 13 x 3 = 39 signals; 39 x 42 = 1,638 samples;
- Study B: 17 x 3 = 51 signals; 51 x 110 = 5,610 samples
- Total for both studies: 90 signals; 7,248 samples.

Below, we illustrate the application of the methodology (described in the Methods section) to one example signal from Study A and to one from Study B. These example signals, referred to as "Signal A" and "Signal B", are shown in Figure 2.

3.2 Application of Methodology to Signal A & Signal B

We present here the results of applying the methodology to Signal A and Signal B. Specifically, we show the results of applying the three techniques for checking the normality condition (histogram, Q-Q plot, and S-W test) and the one technique for checking the independency condition (linear regression).

3.2.1 Check of Normality Condition

Figure 3(a) shows the histogram of log-ratio sample values - or, simply, log-ratios - for Signal A, and the corresponding "best-fit" normal (i.e. Gaussian) PDF, i.e. that with the mean and standard deviation of the log-ratios. Figure 3(b) shows the corresponding subfigure for Signal B.

Figure 4(a) shows the Q-Q plot of log-ratios for Signal A, and the corresponding "best-fit" straight line, i.e. that minimizing the total least-square fit error. Figure 4(b) shows the corresponding subfigure for Signal B.

Since the histogram and Q-Q plot techniques are graphical, it is by visual inspection of the plots that one must decide whether the normality condition is verified or not. Through our experience based on looking at tens of such plots in the particular context of LoD signals, as well as at many in the literature for other applications, we conclude with confidence that, according to the histograms and Q-Q plots, Signal A and Signal B both satisfy the normality condition.

The application of the S-W test with significance level $\alpha = 0.05$ leads to the following conclusions.

For Signal A: The p-value of the S-W test is 0.10. Since it is greater than 0.05, we cannot reject the H_0 hypothesis that the log-ratio RVs have a normal (i.e. Gaussian) PDF. Recall that this does NOT allow us

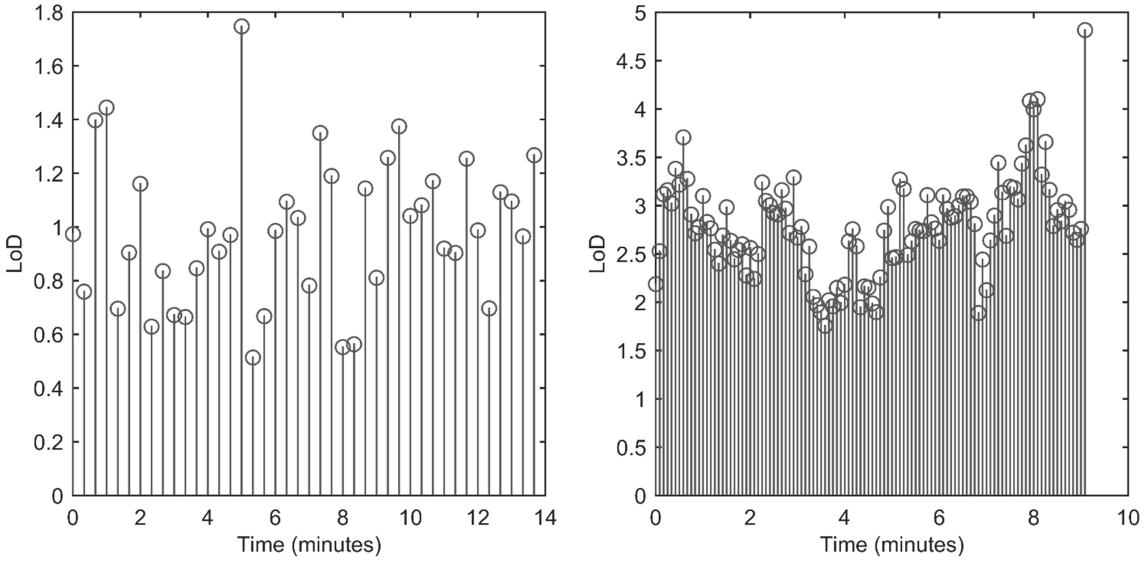

Figure 2: Example LoD signals from (a) Study A and (b) Study B, referred to as ”Signal A” and ”Signal B”, respectively.

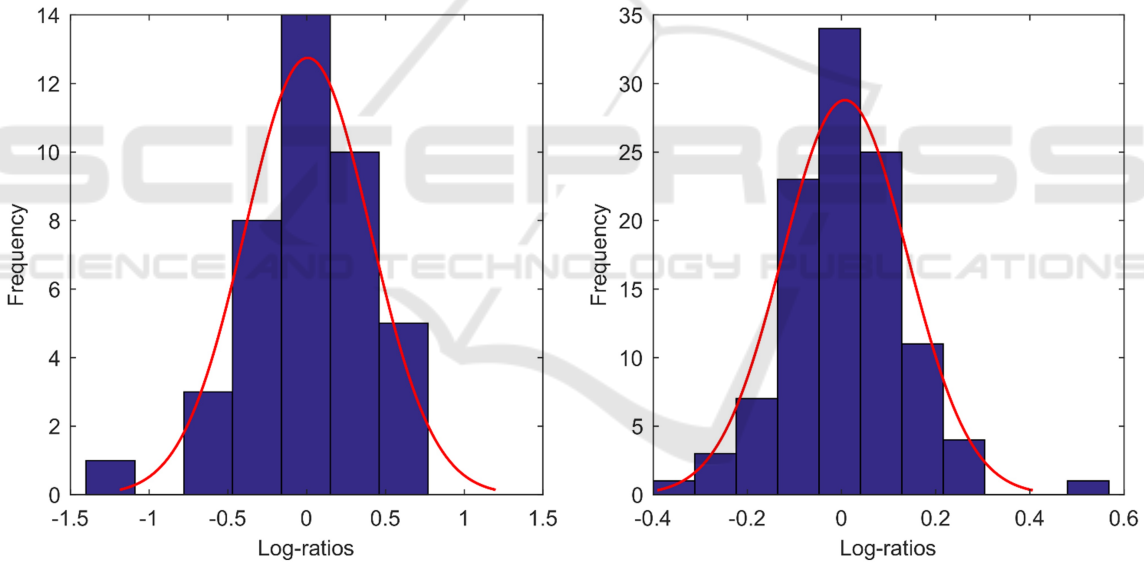

Figure 3: Histograms of log-ratio sample values (or log-ratios) for (a) Signal A and (b) Signal B, and corresponding ”best-fit” normal PDFs.

to conclude that Signal A then verifies the normality condition.

For Signal B: The p-value of the S-W test is 0.01. Since it is smaller than 0.05, we must reject the H_0 hypothesis. This allows us to conclude that Signal B does not verify the normality condition according to the present test.

The results of the three techniques (histogram, Q-Q plot, and S-W test) lead us to the following conclusions.

For Signal A: The pair of graphical tests indicates that Signal A satisfies the normality condition, and the S-W test does NOT allow us to say that it does not satisfy this condition, which is the best one could hope for in light of the two other tests.

For Signal B: The pair of graphical tests indicates that Signal B satisfies the normality condition, whereas the S-W test indicates that it does not.

While all three techniques allow us to conclude with confidence that Signal A verifies the normality

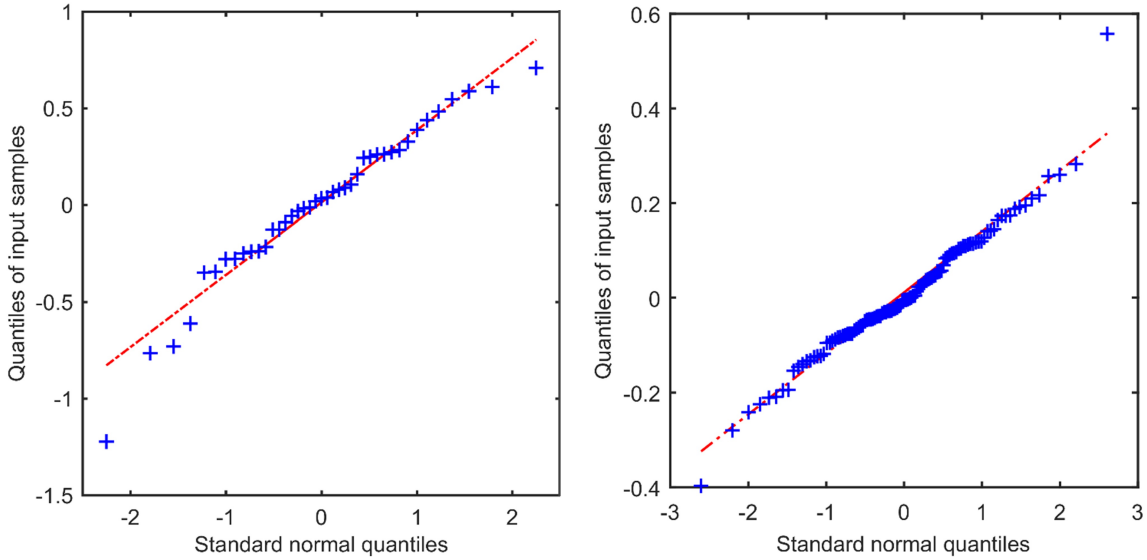

Figure 4: Q-Q plots of log-ratio sample values (or log-ratios) for (a) Signal A and (b) Signal B, and corresponding ”best-fit” straight lines.

condition, one cannot so easily reach a conclusion for Signal B. However, since the graphical techniques both show in a convincing way that Signal B verifies the normality conditions, we made the decision to declare that Signal B verifies the normality condition.

3.2.2 Check of Independency Condition

Figure 5(a) shows the plot of log-ratio sample values (or log-ratios) for Signal A as a function of time, and the corresponding ”best-fit”/regression straight line. Figure 5(b) shows the corresponding subfigure for Signal B.

Since the linear regression technique is graphical, it is by visual inspection of the plots that one must decide whether the independency condition is verified or not. As for the histograms and Q-Q plots, through our experience of looking at many such plots, we conclude with confidence - from the regression lines - that Signal A and Signal B both verify the independency condition.

3.2.3 Conclusion for Signal A & Signal B

Since each of Signal A and Signal B can reasonably be said to verify both the normality condition and the independency condition, we conclude that GBM is a valid choice of model for each of Signal A and Signal B.

3.3 Results for all 90 LoD Signals

We applied to all 90 LoD signals available the same detailed analysis as the one applied above for Signal A and Signal B.

3.3.1 Check of Normality Condition

According to the graphical techniques of histogram and Q-Q plot, we concluded that all 90 LoD signals from Study A and Study B verify the normality condition.

According to the S-W test with significance level $\alpha = 0.05$, we reached the following conclusions.

For Study A: for each LoD signal in this study, the H_0 hypothesis cannot be rejected. Recall that this does NOT allow us to conclude that such signal then verifies the normality condition.

For Study B: for each LoD signal in this study, the H_0 hypothesis must be rejected. This allows us to conclude that such signal does NOT verify the normality condition according to the present test.

Therefore, for Study A, all LoD signals verify the normality condition according to all three techniques, whereas, for Study B, they verify it only according to the graphical techniques.

However, just as in the case of Signal B above, the graphical techniques of histogram and Q-Q plot allowed us to conclude that all LoD signals in Study B can reasonably be said to verify the normality condition.

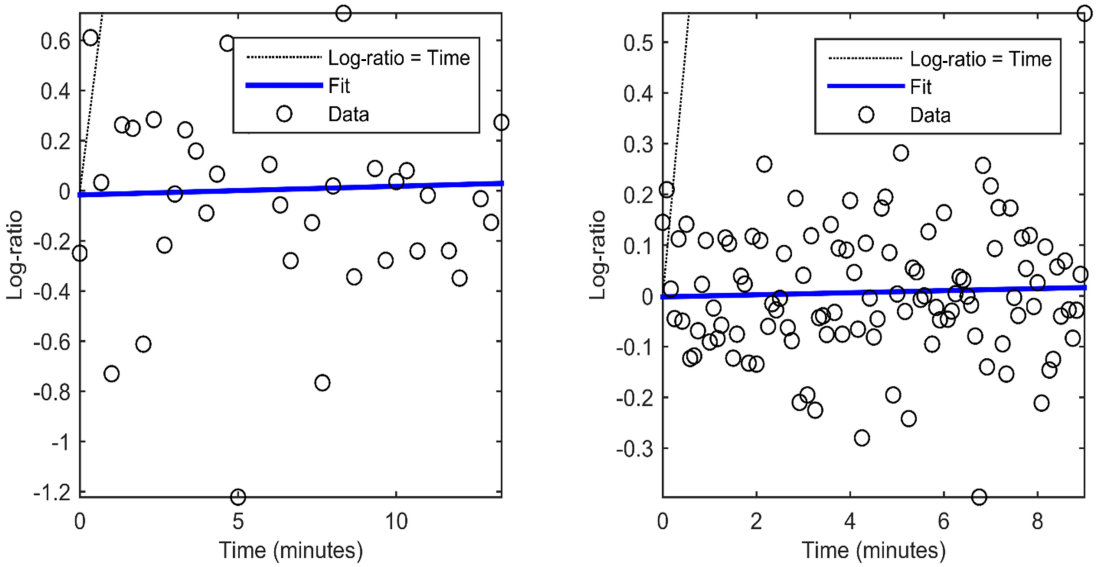

Figure 5: Plots of log-ratio sample values (or log-ratios) as a function of time and of the corresponding "best-fit" straight lines for (a) Signal A and (b) Signal B.

In conclusion, all 90 LoD signals from Study A and Study B can reasonably be said to verify the normality condition.

3.3.2 Check of Independency Condition

The linear regression technique allowed us to declare that all LoD signals from Study A and Study B verify the independency condition.

3.3.3 Conclusion for all LoD Signals from Study A & Study B

Since each signal in Study A and in Study B can reasonably be said to verify the normality condition and the independency condition, we conclude that GBM is a valid choice of model for each signal in Study A and in Study B.

4 DISCUSSION

Using a carefully planned and executed protocol, and a validated drowsiness monitoring system, we collected discrete-time signals that represent the evolution of the level of drowsiness (LoD) of 30 individuals at three increasing levels of sleep deprivation over three days, thus resulting in 90 validated LoD signals.

Given their nature, LoD signals must be viewed as realizations of random processes (RPs). Predicting the (unknown) future values of such signals, as well as related events, based upon data available up to close to the present time requires one to have a model of the underlying RP.

Using appropriate statistical analysis, we have shown that Geometric Brownian Motion (GBM) is a valid choice of random process (RP) model for modelling all 90 LoD signals, thus without a single exception.

Of course, before being sure that GBM is an excellent model for "all" LoD signals to be encountered in all real-life circumstances, it will be necessary to conduct studies on more subjects and in actual operational conditions such as driving on a real road.

At the time of this writing, we are exploring the avenues that open up in the field of drowsiness monitoring if/when the LoD signals are GBM.

In addition, the work reported here can provide a strong motivation for researchers interested in modelling physiological and biological processes to examine whether - starting from the physiological and biological mechanisms involved in the evolution of drowsiness, and using appropriate mathematical models and tools - they can possibly show that an LoD signal should in fact naturally be a realization of an underlying GBM RP model.

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