Wastewater Treatment Plant Design: Optimizing Multiple Objectives

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Abstract: The high costs associated with the design of wastewater treatment plants (WWTPs) motivate the research in the area of modelling their construction and the water treatment process optimization. This work addresses different methodologies, which are based on defining and simultaneously optimizing several conflicting objectives, for finding the optimal values of the state variables in the plant design. We use an evolutionary many-objective optimization algorithm with clustering-based selection that proved effective in handling challenging optimization problems with a large number of objectives. The obtained results are promising and with physical meaning. It is shown that the overall WWTP design can be improved by coming up with appropriate formulation of the optimization problem and solving approach.

1 INTRODUCTION

The high costs associated with the design and operation of wastewater treatment plants (WWTPs) motivate the research in the area of WWTP modelling and the water treatment process optimization. The WWTP design involves optimizing simultaneously several conflicting objectives, for finding the optimal values of the state variables. These problems are called multiobjective optimization problems.

Without loss of generality, a multiobjective optimization problem with m objectives and n decision variables can be formulated mathematically as follows:

$$\begin{aligned} \text{minimize:} \quad & \mathbf{F}(\mathbf{x}) = (f_1(x), \ldots, f_m(x))^{\mathrm{T}} \\ \text{subject to:} \quad & \mathbf{g}(\mathbf{x}) \leq 0 \\ & \mathbf{h}(\mathbf{x}) = 0 \\ & \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where $\mathbf{x}$ is the decision vector defined in the decision space $\Omega = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}\}$, $\mathbf{l}$ and $\mathbf{u}$ are the lower and upper bounds of the decision variables, respectively, $\mathbf{g}(\mathbf{x})$ is the inequality constraints vector, $\mathbf{h}(\mathbf{x})$ is the equality constraints vector, and $\mathbf{F}(\mathbf{x})$ is the objective vector defined in the objective space $\mathbb{F}^m$. When $m > 3$, the problem in (1) is often referred to as many-objective optimization problem.

The presence of multiple objectives makes the objective space partially ordered. For comparing different solutions under such circumstances, the concept of the Pareto dominance is commonly used. A solution $\mathbf{a}$ is said to dominate a solution $\mathbf{b}$, denoted as $\mathbf{a} \prec \mathbf{b}$, iff $\forall i \in \{1,\ldots,m\} : f_i(\mathbf{a}) \leq f_i(\mathbf{b})$ and $\exists j \in \{1,\ldots,m\} : f_j(\mathbf{a}) < f_j(\mathbf{b})$. Opposing single-objective optimization, the solution to multiobjective optimization problems is not a single solution, but a set of non-dominated solutions called the Pareto-optimal set. A solution $\mathbf{a}$ is called Pareto optimal, iff $\nexists \mathbf{b} \in \Omega : \mathbf{b} \prec \mathbf{a}$, or there is no feasible solution $\mathbf{b}$ such that $\mathbf{b}$ dominates $\mathbf{a}$. The main goal of the multiobjective optimization is to obtain the set of Pareto optimal solutions.

In this work, two distinct approaches to the WWTP design optimization problem are proposed. In the first approach, the WWTP is optimized in terms of the secondary treatment by simultaneously minimizing the sum of investment and operation costs and maximizing the effluent quality, in a way that the strict laws on effluent quality are accomplished. The second approach consists of optimizing the WWTP design by simultaneously minimizing the variables that most influence the operation and investment costs as well as the effluent quality. This approach results in a many-objective optimization problem.

Classical methods for solving multiobjective optimization problems mostly rely on scalarization, which means converting multiple objectives into a single objective function that depends on some parameters. The resulting single-objective problem is solved to find a single Pareto optimal solution. Repeated runs with different parameter settings are used

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to find multiple Pareto optimal solutions (Miettinen, 1999).

Alternatively, evolutionary algorithms have emerged as a powerful tool for solving multiobjective optimization problems. Evolutionary algorithms are stochastic problem solving techniques whose working mechanism is based on mimicking the principles of natural evolution. Evolutionary algorithms are particularly suitable for solving multiobjective optimization problems because they simultaneously deal with a set of possible solutions, called population. This feature allows to find several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform series of separate runs that can be infeasible in practice.

Multiobjective evolutionary algorithms (MOEAs) are often differentiated based on the selection mechanism. Dominance-based algorithms rely on the concept of the Pareto dominance typically combined with some diversity preserving technique (Deb et al., 2002). Decomposition-based approaches decompose an original problem into several subproblems and solve them in parallel (Zhang and Li, 2007). Indicator-based algorithms seek to improve the values of quality indicators for the current Pareto set approximations in attempt to direct the search (Zitzler and Künzli, 2004). Despite the success in various application domains, MOEAs often exhibit certain limitations. For instance, dominance-based selection can be ineffective in maintaining diversity (Denysiuk and Gaspar-Cunha, 2017a) and for problems with a large number of objectives. The performance of decomposition-based approaches can be highly influenced by the type of the chosen decomposition scheme (Denysiuk and Gaspar-Cunha, 2017b). The applicability of indicator-based algorithms can be limited by a high computational cost.

As this study considers the WWTP design by applying different optimization approaches, including the one requiring the simultaneous optimization of a large number of objectives, it is of critical importance to adopt an appropriate problem solving strategy. Due to exhibited competitiveness on a set of challenging many-objective problems, we use an evolutionary many-objective optimization algorithm with clustering-based selection (EMyO/C) (Denysiuk et al., 2014). In EMyO/C, the Pareto dominance-based selection leads to a set of solutions naturally reflecting the notion of optimality in multiobjective optimization, clustering specifically adapted to many-objective optimization ensures scalability and adequate diversity of solution, whereas a modified differential evolution operator provides an effective exploration of the complex search space of the WWTP model.

2 MATHEMATICAL MODEL

A typical WWTP comprises a primary and secondary treatment of the wastewater. This work focuses solely on the secondary treatment, in particular on an activated sludge system. This system consists of an aeration tank and a secondary settler, which are mathematically described by the ASM1 model (Henze et al., 1986) and the ATV model combined with the double exponential model (Espírito Santo et al., 2006), respectively.

2.1 Mass Balances around Aeration Tank

The first set of equations come from the mass balances around the aeration tank. The Peterson matrix of the ASM1 model (Henze et al., 1986) is used to define the model for the mass balances. This model is widely accepted by the scientific community, as it produces good predictive values by simulation (Alex et al., 2008). For a CSTR (completely stirred tank reactor), it is assumed that the mass of a given component entering the tank minus the mass of the same compound in the tank, plus a reaction term (positive or negative) equals the accumulation in the tank of the same compound:

$$\frac{Q}{V_a}(\xi_{in} - \xi) + r_\xi = \frac{d\xi}{dt}. \quad (2)$$

It is convenient to refer that in a CSTR the concentration of a compound is the same at any point inside the reactor and at the effluent of that reactor. The reaction term for the compound in question, r_ξ, is obtained by the sum of the product of the stoichiometric coefficients, $\nu_{\xi j}$, with the expression of the process reaction rate, ρ_j, of the ASM1 Peterson matrix (Henze et al., 1986)

$$r_\xi = \sum_j \nu_{\xi j} \rho_j. \quad (3)$$

In steady state, the accumulation term given by $\frac{d\xi}{dt}$ is zero, because the concentration of a given compound is constant in time. The ASM1 model involves 8 processes incorporating 13 different components. The mass balances for the inert materials, S_I and X_I, are not considered because they are transport-only components.

The processes involved are aerobic growth of heterotrophs, anoxic growth of heterotrophs, aerobic growth of autotrophs, decay of heterotrophs, decay of autotrophs, ammonification of soluble organic nitrogen, hydrolysis of entrapped organics, and hydrolysis of entrapped organic nitrogen.

The equation obtained from the ASM1 model with mass balances is as follows. For example, to the soluble substrate (S_S)

$$\frac{Q}{V_a}\left(S_{S_{in}} - S_S\right) - \frac{1}{Y_H}\rho_1 - \frac{1}{Y_H}\rho_2 + \rho_7 = 0. \quad (4)$$

The other components are slowly biodegradable substrate (X_S), heterotrophic active biomass (X_{BH}), autotrophic active biomass (X_{BA}), particulate products arising from biomass decay (X_P), nitrate and nitrite nitrogen (S_{NO}), $NH_4^+ + NH_3$ nitrogen (S_{NH}), soluble biodegradable organic nitrogen (S_{ND}), particulate biodegradable organic nitrogen (X_{ND}), alkalinity (S_{alk}), oxygen (S_O), where Y_H is a stoichiometric parameters. For oxygen mass transfer, aeration by diffusion is considered.

2.2 Secondary Settler Constraints

Traditionally, the importance of the secondary settler is underestimated when compared with the aeration tank. However, it plays a crucial role in the activated sludge system. For example, the clarification efficiency of the settling tank has great influence on the treatment plant efficiency because the particulate fraction arising from biomass contributes to the major portion of the effluent *COD*. Further, it has been observed that the investment cost of a typical settling tank in a WWTP context could reach 25% of the total (Zeng et al., 2003). Thus, when trying to reduce both investment and operation costs, the importance of the secondary settler is by far emphasized.

When the wastewater leaves the aeration tank, where the biological treatment took place, the treated water should be separated from the biological sludge, otherwise, the *COD* would be higher than it is at the entry of the system. The most common way of achieving this purpose is by sedimentation in tanks.

The behavior of a settling tank depends on its design and operation, namely the hydraulic features, as the flow rate, the physical features, as inlet and sludge collection arrangements, site conditions, as temperature and wind, and sludge characteristics. The factors that most influence the size of the tank are the wastewater flow and the characteristics of the sludge. As the influent flow is known, the optimization of the sedimentation area and depth must rely on the sludge characteristics, which in turn are related with the performance of the aeration tank. So, the operation of the biological reactor influences directly the performance of the settling tank and for that reason, one should never be considered without the other.

The details of the used settling tank model can be found in (Espírito Santo et al., 2006).

2.3 Composite Variables

In a real system, some state variables are, most of the time, not available from direct measurements. Thus, readily measured composite variables are used instead. We used the particulate chemical oxygen demand, soluble chemical oxygen demand, chemical oxygen demand, volatile suspended solids, total suspended solids, biochemical oxygen demand, total nitrogen of Kjeldahl, and total nitrogen.

2.4 Quality Constraints

Quality constraints are usually derived from environmental law restrictions. The most used are related with limits in *COD*, *N*, and *TSS* at the effluent. In mathematical terms, these constraints are defined as:

$$\begin{aligned} COD_{ef} &\leq COD_{law} \\ N_{ef} &\leq N_{law} \\ TSS_{ef} &\leq TSS_{law} \end{aligned} \quad (5)$$

where the subscript "*ef*" stands for effluent.

2.5 Flow and Mass Balances

The system behavior, in terms of concentration and flows, may be predicted by balances. In order to achieve a consistent system, these balances must be done around the entire system and not only around each unit operation. This is crucial to reinforce the robustness of the model. Furthermore, these balances may not be a sum of the mass balances of the individual components since the PWWF events are contemplated in the ATV design included in the settler modelling. The balances were done to the suspended matter, dissolved matter and flows.

2.6 System Variables Definition

To complete the model, some definitions are added, such as sludge retention time, hydraulic retention time, recycle rate, recycle rate in a PWWF event, recycle flow rate during a PWWF event, and maximum overflow rate. A fixed value for the relation between volatile and total suspended solids was also considered (Espírito Santo et al., 2006).

2.7 Simple Bounds

All variables must be nonnegative, although more restricted bounds are imposed on some of them due to operational consistencies (Espírito Santo et al., 2006).

3 ALGORITHM

The evolutionary many-objective optimization algorithm with clustering-based selection (EMyO/C) starts by randomly generating a population of solutions P and computing the components of a reference point as

$$z_j = \min_{1 \le i \le \mu} f_j(x^i) \tag{6}$$

for $j = 1,\ldots,m$ where μ is a population size.

Thereafter, the population is evolved for a fixed number of generations $g_{\max}$. Each generation embraces producing an offspring population Q in the variation procedure that is merged with P and reduced to the size of μ in the replacement procedure.

The variation procedure relies on a modified differential evolution (DE) operator presented in (Denysiuk et al., 2013). Each individual in P produces single offspring, which can be described as follows. Two different individuals x^1 and x^2 are randomly selected from P. A mutation vector v is computed as

$$v_j = \begin{cases} x^1_j - x^2_j + \delta_j(u_j - l_j) & \text{if } r_j \le p_m \\ x^1_j - x^2_j & \text{otherwise} \end{cases} \tag{7}$$

with

$$\delta_j = \begin{cases} (2u_j)^{\frac{1}{\eta_m+1}} - 1 & \text{if } u_j \le 0.5 \\ 1-(2-2u_j)^{\frac{1}{\eta_m+1}} & \text{otherwise} \end{cases} \tag{8}$$

where r_j and u_j are random numbers from $\mathbb{U}(0,1)$ for $j = 1,\ldots,n$, whereas p_m and η_m are control parameters. The obtained mutation vector is restricted as

$$v_j = \begin{cases} -\rho_j & \text{if } v_j < -\rho_j \\ \rho_j & \text{if } v_j > \rho_j \\ v_j & \text{otherwise} \end{cases} \tag{9}$$

where $\rho_j = (u_j - l_j)/2$ for $j = 1,\ldots,n$. An offspring x' is generated by mutating the parent as

$$x'_j = \begin{cases} x_j + v_j & \text{if } \xi_j \le CR \\ x_j & \text{otherwise} \end{cases} \tag{10}$$

where CR is a control parameter of the DE operator and $\xi_j \sim \mathbb{U}(0,1)$. Lastly, the offspring feasibility is ensured as

$$x'_j = \begin{cases} l_j & \text{if } x'_j < l_j \\ u_j & \text{if } x'_j > u_j \end{cases} \tag{11}$$

for $j = 1,\ldots,n$. The resulting offspring x' is compared with its parent x. If necessary the above described steps - including computation of v, mutation restriction, and creation of x' - are performed until x' differs from x in at least one gene. The offspring is evaluated and the components of the reference point are updated if there are smaller objective values.

The replacement procedure forms the population of the next generation by selecting promising individuals from the multiset composed of the current and offspring populations. The nondominated sorting is performed to sort individuals into different nondominated fronts. In order to handle constraints, the constrained-domination principle (Deb et al., 2002) is used. Under this principle, solution a is said to constrained-dominate solution b, if any of the following conditions is true: (i) solution a is feasible and solution b is not, (ii) solutions a and b are both infeasible, but solution a has a smaller overall constraint violation, (iii) solutions a and b are feasible and solution a dominates solution b. For a given solution, the overall constraint violation cv is calculated as:

$$cv = \sum_{i=1}^{p} \max\{0, g_i(x)\} + \sum_{j=1}^{q} |h_j(x)|. \tag{12}$$

where $g_i(x) \le 0$ for $i = \{1,\ldots,p\}$ and $h_i(x) = 0$ for $j = \{1,\ldots,q\}$ are inequality and equality constraints. Then, each front is added to the new population one at a time until the predefined population size is reached. In case when the last accepted front $\mathcal{F}_l$ cannot be completely accommodated, k best individuals are selected from $\mathcal{F}_l$ as follows.

For each individual in $\mathcal{F}_l$, the objectives are projected onto the unit hyperplane as

$$f_j = \frac{f_j - z_j}{\|f - z\|_1} \ \forall i \in \{1,\ldots,m\}. \tag{13}$$

Using these values, k clusters are formed as follows.

Step 1 Initially, each individual belongs to a separate cluster $C = \{C_1,\ldots,C_{|\mathcal{F}_l|}\}$.

Step 2 If $|C| = k$, stop. Otherwise, go to Step 3.

Step 3 For each pair of clusters, the distance between two clusters d_{12} is calculated as

$$d_{12} = \frac{1}{|C_1||C_2|} \sum_{i \in C_1, j \in C_2} d(i,j) \tag{14}$$

where $d(i,j)$ is the Euclidean distance between individuals i and j.

Step 4 The pair of clusters having the smallest distance is merged. Go to Step 2.

In each cluster, a representative is identified and selected to the new population. Specifically, a cluster representative is an individual having the smallest distance to the reference point among the individuals in the same cluster.

4 MULTIOBJECTIVE APPROACH

In this section, the WWTP design is optimized by minimizing the total cost function and maximizing the effluent quality measured by the quality index function.

4.1 Total Cost Function

The cost function represents the total cost and includes both investment (*IC*) and operation (*OC*) costs. To obtain the cost function based on real data, a study was carried out with a WWTP building company and relevant parameters were estimated by a least squares technique for a basic model (Tyteca, 1976).

The investment cost for the aeration tank is

$$IC_a = 148.6V_a^{1.07} + 7737G_S^{0.62} \tag{15}$$

where V_a is the volume and the air flow G_S.

The operation cost for the aeration tank is

$$OC_a = \left[0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right] 148.6V_a^{1.07} + (1+i)^{-10} 7737G_S^{0.62} + 115.1\Gamma P_c G_S, \tag{16}$$

where the term $(1+i)^{-10}$ is used to bring to present a future value, in this case, 10 years from now, and Γ is a term used to bring this value to the present.

The investment cost for the settling tank is

$$IC_s = 955.5A_s^{0.97}. \tag{17}$$

The operation cost function for the settling tank is

$$OC_s = \left[0.01\Gamma + 0.02\Gamma(1+i)^{-10}\right] 148.6(A_s h)^{1.07}. \tag{18}$$

Lastly, the total cost function (*TC*) is given by the sum of all the previous functions

$$TC = 174.2V_a^{1.07} + 12487G_S^{0.62} + 114.8G_S + 955.5A_s^{0.97} + 41.3(A_s h)^{1.07}. \tag{19}$$

4.2 Quality Index Function

A quality index function is used to measure the amount of pollution in the effluent. It can be useful to attain a required level of effluent quality. The BSM1 model (Alex et al., 2008) defines the quality index (*QI*) by measuring the amount of daily pollution in average terms during seven days

$$QI = (2TSS + COD + 2BOD + 20TKN + 2S_{NO})\, Q_{ef}/1000. \tag{20}$$

where TSS is the total of suspended solids, COD is the chemical oxygen demand, BOD is the biochemical oxygen demand, TKN is the total Kjeldahl nitrogen, S_{NO} is the nitrate and nitrite nitrogen and Q_{ef} is the effluent flow.

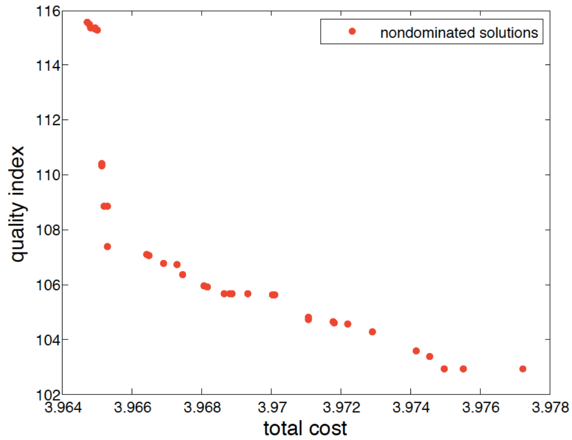

Figure 1: Trade-off curve for the total cost and quality index.

4.3 Results

This approach resulted in optimization problem with 2 objective functions, 115 decision variables being bounded below and above, an inequality constraint and 99 equality constraints. Due to a highly constrained search space, first a feasible solution was found. For this, 30 independent runs of EMyO/C were performed setting $\mu = 1000$, $g_{\max} = 1000$, $CR = 0.15$, $\eta_m = 20$ and $p_m = 1/n$. Then, the best found solution among all the runs was improved by a single-objective optimizer, HGPSAL (Espírito Santo et al., 2010). In order to obtain a set of Pareto optimal solutions, another 30 independent runs of EMyO/C were performed setting $\mu = 300$, $g_{\max} = 300$ and introducing the obtained feasible solution into the initial population.

Figure 1 depicts all nondominated solutions obtained after the performed experiments. The values of the *TC* are in millions of euros (M €). In this figure, the compromise solutions representing trade-offs between the total cost (*TC*) and quality index (*QI*) are plotted. It can be seen that the proposed approach to the WWTP design produces results with a physical meaning. Specifically, the lower values of the quality index can be achieved through the increase in the total cost, while the smaller values of the total cost result in the larger amount of pollution in the effluent, measured by the quality index.

In Table 1, the objective values along with the most important decision variables of the obtained trade-off solutions are presented, namely, the aeration tank volume (V_a), the air flow rate (G_S), the sedimentation area (A_s), the settler tank depth (h), the total nitrogen (N), the chemical oxygen demand (COD), and the total suspended solid (TSS). The presented solutions are obtained after applying clustering pro-

Table 1: Optimal values for the most important variables obtained using multiobjective approach.

V_a	G_S	A_s	h	N	COD	TSS	TC(M €)	QI
100	6597.8531	271.7075	1	1.8982	125	10.7124	3.9647	115.5605
100	6598.438	271.7075	1	1.8982	125	10.4436	3.9649	115.3593
100	6598.9164	271.7075	1	1.3069	125	11.7056	3.9651	110.4133
100	6599.3342	271.7075	1	1.0247	125	11.7056	3.9653	108.8466
100	6599.363	271.7075	1.0637	0.90931	125	10.8241	3.9664	107.1068
100	6599.363	271.7075	1.0906	0.77282	125	10.8241	3.9669	106.7584
100	6599.363	271.7075	1.1116	0.83292	125	10.9469	3.9673	106.7376
100	6599.363	271.7075	1.1208	0.77282	125	10.8241	3.9674	106.3607
100	6599.363	271.7075	1.1598	0.71443	125	10.8241	3.9681	105.9032
100	6599.363	271.7075	1.1876	0.64562	125	10.82	3.9686	105.6695
100	6599.363	271.7075	1.1961	0.64562	125	10.82	3.9688	105.6695
100	6599.363	271.7075	1.2245	0.64562	125	10.82	3.9693	105.6695
100	6599.363	271.7075	1.2649	0.64562	125	10.7742	3.97	105.6352
100	6599.363	271.7075	1.3209	0.53704	125	10.8241	3.971	104.7857
100	6599.363	271.7075	1.3631	0.55629	125	10.8241	3.9718	104.6122
100	6599.363	271.7075	1.3835	0.55629	125	10.8241	3.9722	104.5415
100	6599.363	271.7075	1.4222	0.50336	125	10.8241	3.9729	104.2785
100	6599.363	271.7075	1.4924	0.32773	125	10.8241	3.9742	103.5757
100	6599.363	271.7075	1.5123	0.32773	125	10.8241	3.9745	103.3636
100	6599.363	271.7075	1.5359	0.32773	125	10.8241	3.9749	102.9405
100	6599.363	271.7075	1.5661	0.32773	125	10.8241	3.9755	102.9405
100	6599.363	271.7075	1.6586	0.34713	125	10.9389	3.9772	102.9399

cedure to group the data in the objective space. This way, each solution represents a distinct cluster, hence a different part of the trade-off curve. This procedure allows to reduce the number of points, thereby facilitating the visualization of the results. Thus, different design perspectives can be easily observed from the obtained solutions.

It can be seen that the aeration tank volume and the sedimentation area maintain the same values in all the presented solutions. Only the slight variation in a few solutions is observed concerning the air flow rate, whereas the settler depth has the larger variation in the values between all the variables composing the cost function. This hints that the lower cost as well as the desirable values of the quality index can be mostly achieved through controlling the settler depth. Furthermore, it can be seen that the chemical oxygen demand is at the highest allowable level (125), whereas the total nitrogen and the total suspended solid are far below the law limits (15 and 35, respectively).

Moreover, there is not a significant difference in the total cost in the various solutions, giving some freedom in the design. For example, the QI in the first part of the solutions decreases significantly for a small increase in the TC.

5 MANY-OBJECTIVE APPROACH

In this section, the WWTP design is addressed by simultaneously optimizing the most influential variables in the cost and quality index functions.

5.1 Results

To avoid the use of cost functions that are time and local dependent, the variables influencing the investment and operation costs of a WWTP in each unit operation are used as separate objectives. The variables that mostly influence the costs associated with the aeration tank are the volume (V_a) and the air flow (G_S). In terms of investment, the first variable influences directly the cost of the construction of the tank, and the second influences the required power of the air pumps. In terms of operation, both variables will determine the power needed to aerate the sludge properly, as well as the maintenance, in terms of electromechanical and civil construction material due to deterioration. Assuming that the settling process is only due to the gravity, the variables that most influence the costs associated with the secondary settler are the sedimentation area (A_s) and the tank depth (h), for obvious reasons.

Concerning the quality index function, each variable on the right-hand side of (20) is considered as a distinct function, thereby adding to the optimization problem six different objectives. In contrast to the previous formulation, this approach resulted in the optimization problem with 10 objective functions. EMyO/C was applied to the resulting problem setting $\mu = 300$, $g_{\max} = 750$ while keeping the remaining parameters as in the previous experiments and introducing the feasible solution into the initial population.

Table 2: Optimal values for the most important variables obtained using many-objective approach.

V_a	G_S	A_s	h	N	COD	TSS	BOD	TKN	S_{NO}	Q_{ef}	TC(M €)	QI
100	6594.3023	271.7075	1	2.8161	124.2268	11.8656	61.1615	2.8064	0.0802	374.6088	3.9633	122.8764
100	6594.3023	271.7075	1	2.8171	124.1447	19.2583	61.0899	2.8064	0.0802	374.6306	3.9633	128.3383
100	6594.3023	271.7075	1	2.8453	124.3486	29.1979	57.9506	2.8064	0.0802	373.5855	3.9633	133.1374
100	6594.3023	271.7075	1	2.9506	124.2268	11.3468	61.2184	2.8064	0.0802	373.7916	3.9633	122.2631
100	6599.363	271.7075	1	3.2074	123.4647	34.0113	60.1686	3.2387	0.0802	373.2369	3.9653	141.1593
100	6589.2416	271.7075	1.3647	2.1974	125	19.7987	60.7109	1.9417	0.0802	373.4868	3.9678	121.9279
100	6594.3023	271.7075	1.3849	3.2028	123.374	25.6511	61.0103	3.2387	0.0802	374.5473	3.9702	135.9887
100	6599.363	271.7075	1.3036	2.4526	124.136	15.4315	60.4597	2.374	0.0802	373.1675	3.9707	121.2809
100	6599.363	271.7075	1.4351	3.2256	122.8556	34.5301	61.0103	3.2387	0.0802	374.0567	3.9731	142.2592
100	6594.3023	271.7075	1.5554	2.958	123.6176	15.4219	61.1504	2.8064	0.0802	373.6879	3.9733	124.9963
100	6599.363	271.7075	1.5061	2.106	124.5326	34.5059	60.7304	1.9417	0.0802	373.3003	3.9744	132.6874
100	6594.3023	271.7075	1.6622	2.1492	124.5326	12.1456	59.9099	2.374	0.0802	374.1559	3.9753	118.8802
100	6599.363	271.7075	1.5554	2.958	122.8556	30.1749	61.1504	2.8064	0.0802	374.2245	3.9753	135.9325
100	6599.363	271.7075	1.5558	2.4022	125	16.2075	60.6728	2.374	0.0802	373.0643	3.9753	122.3076
100	6594.3023	271.7075	1.6879	3.0518	123.6599	19.0348	60.8003	2.8064	0.0802	377.0701	3.9757	128.6042
100	6599.363	271.7075	1.6298	2.8089	122.904	32.9028	61.0551	2.8064	0.0802	373.848	3.9767	137.7822
100	6599.363	271.7075	1.6298	2.958	123.374	31.3109	60.5197	2.8064	0.0802	373.3762	3.9767	136.1953
100	6599.363	271.7075	1.6879	2.9973	123.9122	19.0348	60.8003	2.8064	0.0802	373.27	3.9777	127.4023
100	6589.2416	271.7075	1.9295	3.2703	124.6942	19.2583	61.6308	2.8064	0.0802	373.3365	3.9782	128.504
100	6594.3023	271.7075	1.927	2.8689	124.3001	13.7515	60.7982	3.2387	0.0802	373.5766	3.9802	126.933

The obtained set of nondominated solutions was reduced by the clustering procedure applied in the objective space. Table 2 presents the most important variables and the objective function values for a representative of each cluster. Additionally, the values of the total cost (TC) and the quality index (QI) were calculated for these solutions. From the table, it can be seen that the values of the aeration tank volume, the sedimentation area as well as the nitrate and nitrite nitrogen are constant for all the presented solutions. The values of the chemical oxygen demand are slightly reduced compared with that obtained using the multiobjective approach. On the other hand, the values of the total nitrogen and the total suspended solids are higher than in the previous experiments. However, they are still below the law limits. Relatively small variations can be observed regarding the other variables, namely, the biochemical oxygen demand, the total nitrogen of Kjeldahl and the effluent flow.

Furthermore, the values of the total cost and the quality index for all obtained nondominated solutions were calculated. This way, a more general approach, which only consists in minimizing the influential variables, is available to the particular case of the design of a WWTP facility. As a result, only four nondominated solutions were obtained in the space defined by TC and QI. These solutions along with all nondominated solutions obtained using multiobjective approach are shown in Figure 2. It is interesting to note that the range of the trade-off curve obtained in the previous experiments was extended. The solution with the smallest cost function value was obtained using the many-objective approach. This can be explained by the fact that in a high dimensional objective space the number of nondominated solutions is usually larger than in smaller dimensional spaces. This effect results in the propagation of the dissimilarities between solutions in the decision space. Typically, this is an undesirable feature that adversely affects the performance of evolutionary multiobjective optimization algorithms. However, the advantageous side effect is that different regions of the search space can be explored, which happens to be the case in the herein described experiments. Thus, the obtained results show that the many-objective approach can be useful not only during a draft project when the exact data about design of a particular WWTP is not available, but also for finding optimal values in the later

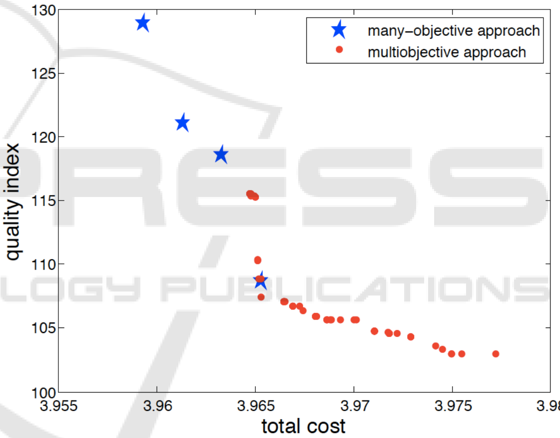

Figure 2: Trade-off curves obtained using multiobjective and many-objective approaches.

stages of the decision-making process. Defining and simultaneously optimizing different combinations of objectives can enrich the final set of the obtained alternatives, which are eventually presented to the decision maker.

6 CONCLUSIONS

This study addressed the optimization of a WWTP whose model parameters were estimated using real Portuguese data. Two different approaches that can be extended to any WWTP unit operation modelling were considered. In the first approach, the WWTP design is addressed through a biobjective optimization problem that consists of minimizing the total cost and the quality index functions. The second approach is more suitable for a draft project, when the exact location and time where the WWTP is to be built is still unknown. This approach involves the simultaneous minimization of the influential variables, resulting in a ten-objective optimization problem.

The results obtained in this study clearly show that the WWTP design can be effectively approached by simultaneously optimizing multiple objectives naturally associated with the problem. The achieved optimal solutions are meaningful in physical terms and are economically attractive. Investment and operation costs are highly influenced by the optimized variables, stressing the importance of optimization. Both approaches to WWTP design provide a set of nondominated solutions that can be used to gain valuable insights about the problem and possible decision making alternatives. This information can help to elaborate a first version of the project and refine important design choices when the specific location and moment in time are defined.

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