Modeling Ethno-social Conflicts based on the Langevin Equation with the Introduction of the Control Function

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Abstract: In this article, we propose a model of ethno-social conflict based on diffusion equations with the introduction of the control function for such a conflict. Based on the classical concepts of ethno-social conflicts, we propose a characteristic parameter - social distance that determines the state of society from the point of view of the theory of conflict.A model based on the diffusion equation of Langevin is developed. The model is based on the idea that individuals interact in society through a communicative field - h. This field is induced by every person in a society, serves as a model of the information interaction between individuals. In addition, the control is introduced into the system through the dissipation function. A solution of the system of equations for a divergent diffusion type is given. Using the example of two interacting-conflicting ethnic groups of individuals, we have identified the characteristic patterns of ethno-social conflict in the social system and determined the effect the social distance in society has in development of similar processes with regard to the external influence, dissipation, and random factors. We have demonstrated how the phase portrait of the system qualitatively changes as the parameters of the control function of the ethno-social conflict change. Using the analysis data of the resulting phase portraits, we have concluded that it is possible to control a characteristic area of sustainability for a social system, within which it remains stable and does not become subject to ethno-social conflicts.

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1 INTRODUCTION

Ethno-social conflicts are a type of social conflict that can be defined as a peak stage in the development of contradictions between individuals, groups of individuals, and society as a whole, which is characterized by the existence of conflicting interests, goals, and views of the subjects of interaction. Conflicts may be hidden or explicit, but they are always based on the absence of compromise, and sometimes even a dialogue between two or more parties (Dollard et al., 1993).

Ethno-social (interethnic) conflict itself can be defined as a kind of relationship between national/cultural groups of individuals characterized by a confrontation in an open or latent phase (i.e. from mutual claims to direct military or terrorist actions). Studies on ethno-social conflicts are widely represented both in classical and modern works: (Perov, 2014; Malkov, 2004; Mason, 2013; Castellano et al., 2009; Smith et al., 2013; Traud et al., 2011)

The development of general conflictology at the present stage was significantly influenced by the works of international scientists, who had laid the theoretical foundation for solving specific problems of a complex interdisciplinary science. These are the classic works of L. Coser, R. Dahrendorf, J. Habermas, H. Becker, A. S. Akhiezer (Coser, 2000; Darendorf, 1994), who substantiated the naturalness, attributive character of ethno-political conflicts and their functions in the life of society, K. Boulding, L. Coser, P. Bourdieu (Boulding, 1969), who laid the foundations for the construction of a general theory of conflicts, J. Burton (Davydov, 2008) and his followers, who turned to the problems of effective practical technologies of the settlement and the fundamental resolution of conflicts as a priority for ensuring the effectiveness of conflictology, P. Sztompka (Perov, 2014), who absolutized the "Western, mainstream" path of social salvation, F. Glasl, who proposed modern conflict resolution mechanisms (Kravchenko, 2003).

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In fact, given the significant impact of such phenomena on the society and on the processes associated with it, the methods and ways for describing and predicting ethno-social conflicts are extremely important.

One of the directions for finding solutions to this problem is the prediction and description of social conflict by means of mathematical modeling (Malkov, 2009; Shabrov, 1996; Blauberg and Yudin, 1973; Saati and Kerns, 1991; Bloomfield, 1997; Plotnitskiy, 2001).

Mathematical modeling based on nonlinear dynamics, so widely used in natural science, is still applied quite rarely in sociological research.

In recent years, significant progress has been made in the development of models of social and political processes (Abzalilov, 2012).

The models available to date can be divided into three groups:

1) models - concepts based on the identification and analysis of common historical patterns and their representation in the form of cognitive schemes that describe the logical connections between various factors that affect historical processes (J.Goldstein, I. Wallerstein, L.N. Gumilev, N.S. Rozov and others). Such models generalize the subject matter to a high degree, but they are not of a mathematical, but of purely logical, conceptual nature;
2) special mathematical models of imitative type, created for the description of specific historical events and phenomena (Yu.N. Pavlovsky, L.I. Borodkin, D. Meadows, J.Forrester, et al.). Such models focus on careful registration and description of the factors and processes that affect the phenomena under consideration. Applicability of such models, as a rule, is limited by a rather narrow space-time interval; they are "tied" to a specific historical event and they cannot be extrapolated for extended periods of time;
3) mathematical models, which are intermediate between the two-abovementioned types. These models describe a certain class of social processes without claiming to provide a detailed description of the features for each particular historical event. Their task is to identify the basic regularities characterizing the course of the processes of the discussed type. In this regard, these mathematical models are called basic models (Plotnitskiy, 2001).

Holyst J.A., Kacperski K., Schweiter F. propose a convenient model of public opinion, which views the interaction between individuals as a Brownian motion (Holyst et al., 2000).

However, mathematical modeling based on nonlinear dynamics, so widely used in natural science, is still applied quite rarely in sociological research.

2 PARAMETRIZATION OF ETHNO-SOCIAL CONFLICT

It is important to identify a parameter determinant to an ethno-social conflict, which will underlie the model we are creating. It is clear that this parameter should be logically justified within the framework of the main modern concepts of social conflict.

This parameter is social distance. Previous works (Petukhov et al., 2016) discuss this matter in more detail; therefore, here we will only provide the following provisions critical for understanding of this model:

1. A major social conflict, as a rule, is accompanied by an informational and social distance between individuals and groups of individuals. Such a distance can be based on interethnic, cultural, religious, and economic differences. There can be various reasons for such a conflict: different levels of aggression of social and ethnic groups, contradicting cultural and economic aspirations, etc. Thus, the social-informational distance itself does not cause the conflict, but, as a rule, accompanies it.
2. This distance increases during the course of the conflict, especially in its extreme variants (revolutions, civil wars, etc.), leading the opposing parties to the position of "non-reconciliation". The history, unfortunately, has very few examples of short and medium-term positive scenario for such situations.
3. Therefore, this point of no return, as a rule, occurs just before the onset of the conflict, and such a transition of a social system from one state to another become decisive (triggering) for the overall situation.

In this case, as a rule, very few conflicts in a modern globalizing world occur without external influence and even interference. This raises the question of introducing control into a model of conflict. This control can play a decisive role in its generation and dynamics.

3 FUNDAMENTALS OF THE MODEL

Socio-political processes are subject to constant changes and deformations, therefore from the point of view of mathematical modeling they cannot be set with a high degree of precision. Here we can trace the analogy with the Brownian particle, i.e. a particle that seemingly moves along a rather defined trajectory, but under close examination, this trajectory turns out to be strongly tortuous, with many small knees (Petukhov et al., 2016; Gutz and Korobitsyn, 2000). These small changes (fluctuations) are explained by the chaotic motion of other molecules. In social processes, fluctuations can be interpreted as manifestations of the free will of its individual participants, as well as other random manifestations of the external environment (Gutz and Korobitsyn, 2000).

In physics, these processes are, as a rule, described by Langevin equation of the stochastic diffusion, which has been applied with relative success for modeling of some social processes as well. For example, the previously mentioned model (Holyst, Kasperski, Schweitger, 2000) is based on the use of this equation.

This approach has several advantages:

1. As it has already been mentioned, the approach allows taking into account the manifestations of the free will of its individual participants, as well as other random manifestations of the external environment for the social system.
2. The behavior of a social system can be calculated, both for its entirety, and for separate individuals.
3. This approach allows identifying some distinctive stable modes of functioning of social systems, depending on various initial conditions.
4. Diffusion equations, as a mathematical apparatus, have been sufficiently validated and studied from the point of view of numerical simulation.

The model is based on the assumption that individuals interact in society through a communicative field - h (a similar concept was introduced in (Holyst et al., 2000), but with another parametrization and another type of initial equations). This field is induced by each individual in society and serves as a model of the information interaction between individuals. However, we should keep in mind that here we are talking about a society, which is difficult to classify as an object in classical physical spatial topology. Objectively, from the point of view of information transfer from an individual to an individual, space in society combines both classical spatial coordinates and additional specific parameters and features. This is caused by the fact that in the modern information world there is no need to be close to the object of influence in order to transmit information to it.

Thus, the society is a multidimensional, social-physical space that reflects the ability of one individual to "reach" another individual with his communicative field, that is, to influence it, its parameters and the ability to move in a given space. Accordingly, the position of the individual relative to other individuals in such a space, among other things, models the level of relationships between them and involvement into the information exchange. The proximity of individuals to each other in this model suggests that there is a regular exchange of information between them, which establishes a social connection. The conflict in such a statement of the problem should be regarded as a variant of the interaction of individuals, or groups of individuals, as a result of which the distance (i.e., social distance xi - xj, where xi and xj are the coordinates in social and physical space, i, j = [1, N], where N is the number of individuals or consolidated groups of individuals) between them is growing rapidly.

Conflict management or various options for conflict mediation (Perov, 2014), from the point of view of modeling, are an additional function that depends at least on the coordinates and affects the overall stability and structure of the social system. There are a number of physical analogies that are similarly influenced by physical systems, for example, a dissipative function that can have different forms in different physical conditions (Malkov, 2009).

4 MATHEMATICAL REPRESENTATION OF THE SYSTEM

The communicative field, as in (Petukhov et al., 2016), is represented by a diffusion equation with a divergent type of diffusion:

$$\frac{\partial}{\partial t} h(x_i, t) = \sum_{j=1}^{N} f(x_i, x_j) \vartheta(x_i, x_j) \bar{\delta}_{(k_s^j + k_c^j),(k_s^i + k_c^i)} + D\big(h(x_i, t) - h(x_i, t_0)\big), \quad (1)$$

where $f(x_i, x_j)$ is a function that describes the interaction between individuals, which is modeled by the classical Gaussian distribution;

$$\vartheta(x_i, x_j) = \frac{1}{\varepsilon\sqrt{\pi}} e^{\frac{-(x_i - x_j)^2}{\varepsilon^2}},$$

Function $\vartheta(x_i, x_j)$ is introduced instead of the delta-function to simplify the process of computer modeling;

$\bar{\delta}_{(k_s^j + k_c^j),(k_s^i + k_c^i)}$ is the inverse Kronecker symbol;

D is the diffusion coefficient describing the propagation of the communicative field.

The movement of an individual in space is described by the Langevin equation:

$$\frac{dx_i}{dt} = u(x_i) + k_c^i k_s^i \left(\sum_{j=1, j\neq i}^{N} \frac{\partial}{\partial x_j} h(x_j, t) \right) + \sqrt{2D}\xi_i(t), \quad (2)$$

$u(x)$ is the control function, which we set as:

$$u(x) = -\frac{x_i}{\tau}$$

where τ is the time of relaxation in the society,

k_c^i - coefficient of social activity of the ith individual or a group of individuals,

k_s^i – coefficient of the scientific and technological progress of the ith individual or a group of individuals,

$\xi_i(t)$ –stochastic force.

We believe that the distinctive parameters of the system can take on values:

$$0 < k_c, k_s, D < 1.$$

In the general case, the following are chosen as the initial conditions for equations (1) and (2):

$$x_i|_{t=0} = x_{0i},$$
$$h(x_i, t = 0) = h_{0i}.$$

5 APPROXIMATE SOLUTION OF THE SYSTEM

Let us consider a model of two interacting consolidated ethnic groups of individuals, presumably in a state of conflict. In this case, equations (1) and (2) produce four equations that fully describe the model of interaction of individuals:

$$\begin{cases} \frac{\partial h(x_1,t)}{\partial t} = D[h(x_1,t) - h(x_1,0)] + \alpha k_c^2 k_s^1 e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2}, \\ \frac{\partial h(x_2,t)}{\partial t} = D[h(x_2,t) - h(x_2,0)] + \alpha k_c^1 k_s^2 e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2} \\ \frac{dx_1}{dt} = u(x_1) + k_c^1 k_s^1 \frac{\partial h(x_2,t)}{\partial x_2} + \sqrt{2D}\xi_1(t), \\ \frac{dx_2}{dt} = u(x_2) + k_c^2 k_s^2 \frac{\partial h(x_1,t)}{\partial x_1} + \sqrt{2D}\xi_2(t), \end{cases} \quad (3)$$

where:

$$\psi = k_c^1 + k_s^1 + k_c^2 + k_s^2, \alpha = \frac{1}{\psi\sqrt{\pi}} \bar{\delta}_{k_c^1+k_s^1, k_c^2+k_s^2}.$$

Here, as in (Petukhov et al., 2016): in order to obtain approximate analytic solutions of the system (3), we use the series expansion accurate to first-order quantities of smallness for $\Delta x = x_i - x_{oi}$ $\Delta t = t - t_o$ difference:

$$h(x_i,t) - h(x_{oi},t_o) \approx \left(\frac{\partial h}{\partial x_i}\right)\Big|_{t=0, x_i = x_{0i}} \Delta x + \left(\frac{\partial h}{\partial t}\right)\Big|_{t=0, x_i = x_{0i}} \Delta t,$$

Then, assuming that the following initial conditions are present:

$$x_{oi} = 0, h(x_{oi}, t_o) = \left(\frac{\partial h}{\partial x_i}\right)\Big|_{t=0, x_i = x_{0i}} = \left(\frac{\partial h}{\partial t}\right)\Big|_{t=0, x_i = x_{0i}} = 1,$$

let us integrate the first two equations of the system (3), and then, using the obtained results and the two latter equations of the system (3), considering the continuity of the corresponding functions, transform the system. Let us then differentiate over time.

Assuming that the stochastic forces for the two groups are the same $\xi_1(t) = \xi_2(t)$.

Then, by introducing new variables:

$$y = x_1 - x_2,$$
$$A = D\left(k_c^1 k_s^1 - k_c^2 k_s^2\right),$$
$$B = 2\alpha \frac{\left(\psi^2 + 1\right)}{\psi^2}\left(k_c^1 k_s^1 k_c^1 k_s^2 + k_c^2 k_s^2 k_c^2 k_s^1\right),$$
$$C = \frac{\psi^2 + 1}{\psi^2},$$

we obtain an equation that looks as follows:

$$\frac{d^2y}{dt^2} = A - H\frac{dy}{dt} + Bye^{-Cy^2}, B > 0, C > 0, H = \frac{1}{\tau}, \quad (4)$$

where A, B, C depend on the parameters: k_s^i, k_c^i, D.
Let us write the equation (4) in the Cauchy form:

$$\begin{cases} \frac{dy}{dt} = z, \\ \frac{dz}{dt} = A - Hz + Bye^{-Cy^2}. \end{cases} \quad (5)$$

The system (5) can be viewed as a dynamic system that describes the process of interaction of two individuals or groups of individuals. This system is non-conservative, but finding its equilibrium states is reduced to solving the same system of equations as in the conservative case, see (Petukhov et al., 2016):

$$\begin{cases} z = 0, \\ ye^{-Cy^2} = -\frac{A}{B}. \end{cases} \quad (6)$$

It was shown in (Petukhov et al., 2016) that the corresponding system has two equilibrium states: the saddle and the center. The general theory of dynamical systems states that the saddle is a rough equilibrium state, that is, its type does not change after a sufficiently small change in the system. While the center is a non-rough state of equilibrium, with small changes in the system, such a state of equilibrium shifts to a stable or unstable focus.

Taking into account the discussion of rough and non-rough equilibrium states, it is easy to construct a phase portrait of the system under consideration in the presence of dissipation (Figure 1. Considering the above, the equilibrium state O_2 of the saddle type does not change its type, but the stable separatrix loop will break, while the equilibrium state O_1 of the center type $\tau > 0$ $(H > 0)$ will shift into a steady focus.

Figures 1-6 show phase portraits for the case of two equilibrium states under conditions

$$0 < -\frac{A}{B} < \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A < 0 \quad (7)$$

or conditions

$$-\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}} < -\frac{A}{B} < 0, A > 0 \quad (8)$$

for three different values of the parameter τ $(\frac{1}{2}; 1; 2)$, where $\dot{y} = \frac{dy}{dt}$.

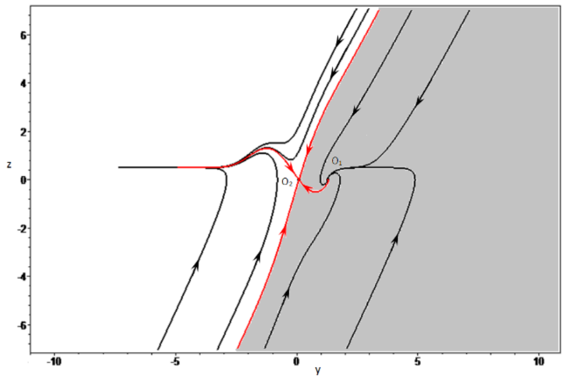

Figure 1: Phase trajectories under conditions (7) and $\tau = \frac{1}{2}$.

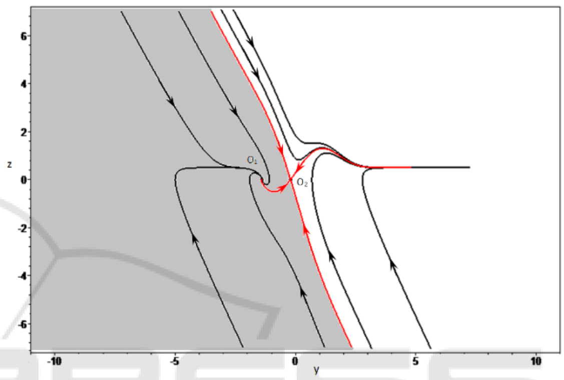

Figure 2: Phase trajectories under conditions (8) and $\tau = \frac{1}{2}$.

The obtained phase portraits show that O_1 is the stable node and O_2 is the saddle. Separatrix, which passes along the border between the gray and white parts of the Figure, refers to the saddle O_2. These separatrices divide the phase plane into areas with qualitatively different behavior of the phase trajectories. The area highlighted in gray is the area of asymptotic stability of the node (region of attraction).

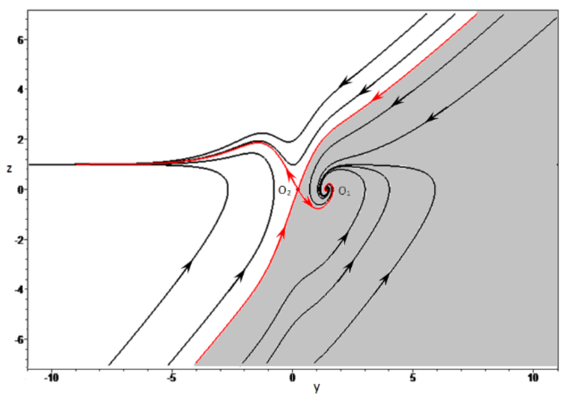

Figure 3. Phase trajectories under conditions (7) and $\tau = 1$.

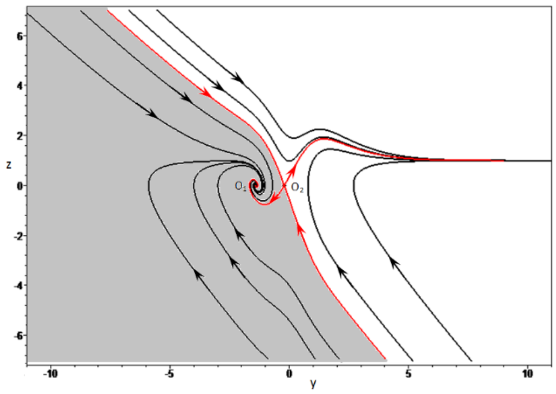

Figure 4: Phase trajectories under conditions (8) and $\tau = 1$.

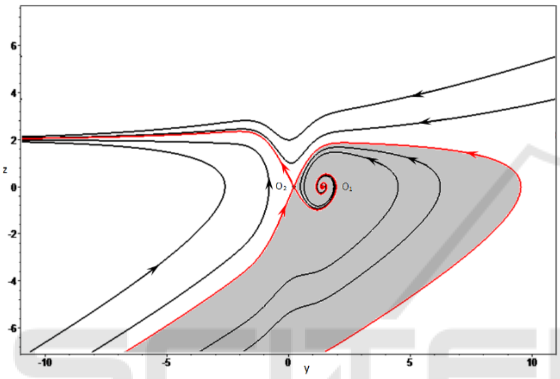

Figure 5: Phase trajectories under conditions (7) and $\tau = 2$.

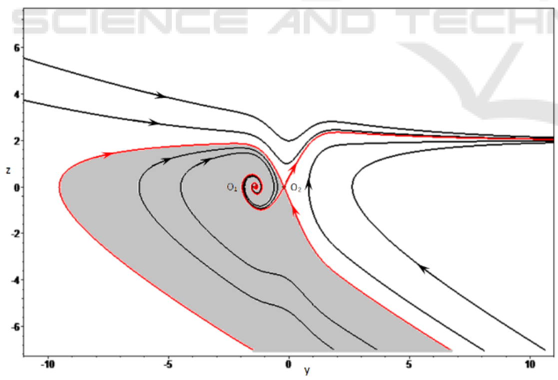

Figure 6: Phase trajectories under conditions (8) and $\tau = 2$.

The obtained phase portraits (Figure 3 – Figure 6) show that O_1 is the steady focus and O_2 is the saddle. Trajectories between the gray and white "zone" of the Figure are the separatrices of the saddle O_2. The gray area is the region of asymptotic stability of the node (region of attraction).

The analysis of the obtained results (see Figure 1, Figure 6) can lead to the conclusion that there is a certain region of asymptotic stability. Figure 4-6 shows that with increasing of the parameter τ (which, from the physical point of view, corresponds to a decrease in the impact of the dissipation function), the region of attraction of the stable equilibrium state decreases, which agrees with the classical concept of the coefficient of friction. The edges of this region are the separatrices of the saddle O_2.

When changing the parameters, we can easily notice that the behavior of the phase trajectories also changes. This concerns purely quantitative changes in the size and location of trajectories, but can lead to significant, qualitative changes in the structure of the phase portrait, i.e. bifurcation. For example, under the following conditions:

$$0 < -\frac{A}{B} < \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A < 0,$$

we have (Figure1 – 6) two simple singular points O_1 and O_2 , but when we reach the value:

$$-\frac{A}{B} = \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A < 0,$$

equilibrium states O_1 and O_2 merge, forming one complex singular point, which, if the following conditions are met:

$$-\frac{A}{B} > \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A < 0,$$

does not appear at all. Thus, bifurcation here is characterized by the birth and disappearance of equilibrium positions. In the model under consideration, the bifurcation values of the parameters are as follows:

$$-\frac{A}{B} = 0, -\frac{A}{B} = \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}},$$

$$-\frac{A}{B} = -\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}.$$

Individuals or groups of individuals, who have the necessary parameters to enter the area of asymptotic stability at the initial moment of time remain at a distance, within which social connections and active information exchange are possible, which means that a conflict state is unlikely or impossible.

As noted in the statement of the task, in a society, where social and informational contact, as well as the interpenetration of different cultures and ethnic groups are sufficient, where separate groups of people do not separate from each other creating closed subsystems (where the conditions differ

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significantly from the basic system), the possibility of the emergence of ethno-social, religious and other conflicts is reduced to a relative minimum.

Individuals or groups of individuals that have fallen outside the region of stability at the initial moment, over time, will end up at a relatively large social distance. This particular state of the social system can be described as the conflict and the manifestation of the existing contradictions between individuals and groups of individuals (Petukhov et al., 2016). For example, in ethno-social conflicts, this is manifested in the minimization of social and cultural contacts between different ethnic groups, the increase in the socioeconomic gap, growing contradictions and, as a result, the transition to an open confrontation phase with the destabilization of the social and political system as a whole.

The control function for an ethno-social conflict *u(x)* (See (2)) introduced here demonstrates how, with a change in its parameters, the phase portrait, and therefore the state of the social system can be substantially changed. This suggests that with a certain mediation, it is possible to achieve a "larger" stability zone, which will attract a greater number of phase trajectories, which in turn provides a greater chance of maintaining the necessary social distance in order to minimize the chances of an ethno-social conflict.

6 CONCLUSIONS

Social hyper-clusterization of society, sharp division in the information and social environment of the coexistence of individuals, and cultural and interethnic dissociation create ideal conditions for social conflict. The prevention of conflicts in society, the definition of their triggers and the search for the most effective scenarios for their suppression are the important tasks for modern social sciences.

This article briefly reviewed the main approaches to modeling in the social sciences, the problems of determining social conflict and its main concepts. A formalized definition of one of the parameters leading to a conflict in the social system is given.

A mathematical model based on the Langevin equation is proposed, an analytical solution is given in the first approximation for a divergent diffusion type. The function of management (mediation) by conflict is introduced based on the physical analogy - the dissipation function.

Specific trigger conditions that take into consideration the external influence and control were established. These conditions are determined by the parameters of the social system, under which the grounds for the emergence of social conflict and its aggravation are created.

Modeling of the system allowed identifying a distinctive region of stability for the social system, determined by phase trajectories. In this area, the studied objects maintain a relatively short social distance between each other, which is typical for social groups, which are actively interacting and stay in a constant information contact. It has been shown how, depending on the impact of the conflict control function, this region is changing.

By determining and correlating these trigger states with the introduced parametrization of the control function, it is possible to determine the patterns corresponding to certain modern ethno-social conflicts, which makes it possible to use this model as a tool for predicting their dynamics and the formation of resolution scenarios.

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