Dynamic Reliability Analysis of Gear Vibration Response with Random Parameters

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Keywords: Random vibration, Gear nonlinear, Dynamic reliability, Poisson process.

Abstract: In order to study the influence of random parameters on the reliability of gear vibration, firstly, a nonlinear stochastic vibration analysis model of gear 3-DOF gap is established based on Newton's Law. And the random response of gear vibration is simulated by stepwise integration method. Secondly, based on the process transcendental theory, a reliability model for the gear nonlinear vibration system with random parametric is established. The calculation formula of the vibration reliability of gear vibration system with random parameters is deduced and its application range is extended. The comparison of examples shows that the parameter stochastic process has little effect on the vibration reliability of the system when the gear system's response is periodic motion, while the vibration reliability of the system will decrease sharply when the gear system's response is chaotic motion. This study provides a reference and theoretical basis for the control and judgment of the nonlinear vibration of gears with random parameters.

1   INTRODUCTION

There are many nonlinear factors in the gear transmission system, such as the gear meshing stiffness, transmission error, bearing clearance, tooth side gap and so on. These coupling factors will cause the strong nonlinear vibration of the gear system and affect the vibration reliability of the gear system. Studies show (So, P., Ott, E., 1995; Shinbrot T., 1993; Li W.,2012; Zhao W, 2012; Li T., 2011) that the system will change from the periodical response to a chaotic vibration state with chaotic, disorder and aperiodic when the parameters of the gear system changed a little. Generally, the gear system response is not sensitive to the small changes of the initial conditions in the periodic response state, however, slight changes will make the system vibration response produce unpredictable results when the gear's system enters the chaotic state.

As we all known, for the gear system with nonlinear vibration, the change of gear's parameters will cause the system into a chaotic vibration state. Traditionally, chaotic vibration state is avoided by the conventional method (such as Lyapunov and bifurcation method), but its dynamic state still changes due to the randomness of gear's parameters. When the system is in chaotic or near-chaotic state,

random bifurcation and random chaos (Zhao W., 2012) of the gear's system response, which affects the vibration and noise of the gear system and determines the vibration reliability of the gear system (Sun Z., 2011).

In order to avoid the chaotic vibration of the gear system and predict the vibration reliability of the system more accurately, the random process characteristics of various parameters is considered into vibration model, so as to better control or avoid this irregular chaotic vibration characteristics. Based on this issue of gear nonlinear vibiration, the method of calculating the vibrational reliability of gears with random parameters is studied in this paper. And it provides a reference and theoretical basis for the control and judgment of the nonlinear vibration of gears with random parameters.

2 NUMERICAL SIMULATION OF GEAR NONLINEAR VIBRATION SYSTEM

2.1 Model of gear nonlinear vibration model with random parameters

To simulate the gear's nonlinear vibration with random parameters, the random parameter is expressed as the combination of the determined value and the disturbed value. For example, the excitation frequency is equivalent to $\omega_m + \omega_{m\Delta}$, where ω_m is the determined value of the excitation frequency, $\omega_{m\Delta}$, is the disturbed value of the excitation frequency. And all parameters are assumed as independent random variable in each time period, namely, the dynamic response of gear vibration is regarded as a Gaussian random process. The three-degree-of-freedom nonlinear coupled dynamic model (as shown in Fig. 1, the specific derivation is shown in Ref. (Li R., 1997)) is taken as the study object. The static transmission error is obtained the first-order components, and the gear nonlinear vibration model with random parameters can be expressed as
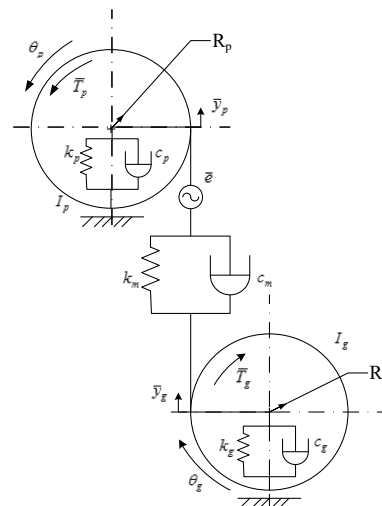
$$
\begin{cases}
\dot{y}_1 = y_2 \\
\dot{y}_2 = -2\xi_p \dfrac{dy_p}{dt} - 2\xi_{mp} \dfrac{dy}{dt} - k_{11}f_p(y_p) - k_{13}f_m(y) \\
\dot{y}_3 = y_4 \\
\dot{y}_4 = -2\xi_g \dfrac{dy_g}{dt} + 2\xi_{mg} \dfrac{dy}{dt} - k_{22}f_g(y_g) + k_{23}f_m(y) \\
\dot{y}_5 = y_6 \\
\dot{y}_6 = F - (\omega_m + \omega_{m\Delta})^2 \cdot \dfrac{e_1}{b}\sin\left[(\omega_m + \omega_{m\Delta})t\right] \\
\qquad + \dot{y}_2 - \dot{y}_4 - 2(\xi_m + \xi_{m\Delta})\dfrac{dy}{dt} - k_{33}f_m(y)
\end{cases} \tag{1}
$$

Figure 1: Dynamic model of a spur gear pair.

in which the dimensionless nonlinear function of gap is expressed as

$$
f_m(y) = \begin{cases}
y - (b_m + b_{m\Delta})/b, & y > (b_m + b_{m\Delta})/b \\
0, & -(b_m + b_{m\Delta})/b \le y \le (b_m + b_{m\Delta})/b \\
y + (b_m + b_{m\Delta})/b, & y < -(b_m + b_{m\Delta})/b
\end{cases} \tag{2}
$$

where ξ_m and $\xi_{m\Delta}$ are the determined value and the disturbed value of meshing damping ratio, respectively；ω_m and $\omega_{m\Delta}$ are the determined value and the disturbed value of excitation frequency, respectively；b_m and $b_{m\Delta}$ are the determined value and the disturbed value of the tooth backlash, respectively；$\xi_{m\Delta}$、$\omega_{m\Delta}$、$b_{m\Delta}$ are similar to Gauss white noise with zero mean.

2.2 Nonlinear vibration numerical solution

For nonlinear random vibration analysis, the most effective method is numerical integration method (Zhao W., 2012; Sun Z., 2011). The numerical simulation of nonlinear random vibration is based on numerical integration. The step-by-step integration method is always used to solve the system dynamics equation, so that the solution of the system in the time domain is obtained. There are many kinds of step-by-step integration methods. At present, linear acceleration method, Runge-Kutta method, Newmark-β method and Wilson-θ method are widely used. In this paper, the Runge-Kutta method is used to solve the dynamic differential equations of the system. The basic steps are:

(1)Determination of the basic random variables and the distribution functions；

(2)Let $t=0$, and give the initial value $x(0), \dot{x}(0)$ ；

(3)Sampling the basic parameters

(4)Establishing dynamic equations of deterministic gear system from sampling results;

(5)Solving the deterministic dynamics equation (4) in the $[t, \Delta t+t]$ moment vibration displacement and velocity by Runge-Kutta method.

3 RELIABILITY ANALYSIS OF NONLINEAR VIBRATION OF GEAR WITH RANDOM PARAMETERS

3.1 Poisson Process Reliability Method Based on Process Leaping

For the random response process, the probability of exceeding the failed span can be respectively expressed as follows (see (Haym B., 2005)).

$$v_Z^+ = \frac{\sigma_{\dot{x}}}{2\pi\sigma_x} \exp\left[-\frac{(Z-\bar{x})^2}{2\sigma_x^2}\right] \tag{3}$$

$$v_Z^- = \frac{\sigma_{\dot{x}}}{2\pi\sigma_x} \exp\left[-\frac{(Z+\bar{x})^2}{2\sigma_x^2}\right] \tag{4}$$

$$v_Z = \frac{\sigma_{\dot{x}}}{2\pi\sigma_x}\left\{\exp\left[-\frac{(Z-\bar{x})^2}{2\sigma_x^2}\right] + \exp\left[-\frac{(Z+\bar{x})^2}{2\sigma_x^2}\right]\right\} \tag{5}$$

Gear vibration reliability and structural dynamic reliability of the first failure beyond the different amplitude of the first time beyond does not mean that the gear structure must gear produce failure. However, due to the periodic meshing of gears, during the meshing cycle of gears, if the amplitude of vibration response of gears exceeds the safety margin for the first time, it will appear in each cycle. If the gear system produces an excessive failure amplitude during one meshing cycle, this amplitude will occur during each meshing cycle, resulting in fatigue failure of the gear.

Therefore, the vibration reliability of gear system can be defined as: the random vibration response $x(t)$ within the gear meshing period $(0, T_D)$ under the random factors does not exceed the maximum safety limit Z_{max} and not less than the minimum safety limit Z_{min}. It can be defined as

$$\begin{aligned} R &= P(Z_{min} \le x(t) \le Z_{max}) \\ &= P(x(t) \le Z_{max}) \cdot P(x(t) \ge Z_{min}) \end{aligned} \tag{6}$$

Where $Z_{max} = \mu_x + Z_{threshold}$, $Z_{max} = \mu_x + Z_{threshold}$ ；

μ_x is the average value of random response in steady response; $Z_{threshold}$ is the safety margin, which is similar to the maximum critical value in the dynamic reliability analysis; and $P(x(t) \le Z_{max})$ and $P(x(t) \ge Z_{min})$ are respectively the probability that the number of times that the positive slope crosses zero during the $(0,t)$ time and the probability that the number of times that the negative slope crosses zero. Assuming that each transcendence is independent and the number of transgressions N obeys Poisson distribution, then the probability of exceeding the number of times n in $(0, t)$ time is

$$p_N(n,t) = \frac{(v\,t)^n e^{-vt}}{n!}, n \ge 0, t \ge 0 \tag{7}$$

Where, v represents the number of transcendence occurred within a unit of time.

(1) The probability of crossing the number of times is zero with positive slope within $t < T_D$ ($P(x(t) \le Z_{max})$)

Considering the characteristics of the Poisson process, if the stochastic process $x(t)$ crosses the positive slope at each pass through Z_{max}, the upper pass rate v_Z^+ is equal to the parameter v. The probability density function of the number of times over the number of N_Z^+ is expressed as

$$p_{N_Z^+}(n,t) = \frac{(v_Z^+ t)^n e^{-v_Z^+ t}}{n!}, n \ge 0, t \ge 0 \tag{8}$$

Then the probability of crossing the number of times is zero with positive slope within $t < T_D$

$$P(x(t) \le Z_{max}) = p_{N_Z^+}(0, T_D) = e^{-v_Z^+ T_D} \tag{9}$$

Substituting Eq.(3) into Eq. (9), then

$$P(x(t) \le Z_{max}) = \exp\left[-\frac{\sigma_{\dot{x}} T_D}{2\pi\sigma_x} \exp\left[-\frac{(Z_{max} - \bar{x})^2}{2\sigma_x^2}\right]\right] \tag{10}$$

(2) the probability of crossing the number of times is zero with positive slope within $t > T_D$ ($P(x(t) \ge Z_{min})$)

If the stochastic process $x(t)$ crosses the negative slope at each pass through Z_{max}, the upper pass rate v_Z^- is equal to the parameter v. The probability density function of the number of times over the number of N_Z^- is expressed as

$$p_{N_{\bar{Z}}}(n,t) = \frac{(v_{\bar{Z}}^- t)^n e^{-v_{\bar{Z}}^- t}}{n!}, n \geq 0, t \geq 0 \qquad (11)$$

Then the probability of crossing the number of times is zero with positive slope within $t > T_D$

$$P(x(t) \geq Z_{\min}) = p_{N_{\bar{Z}}}(0, T_D) = e^{-v_{\bar{Z}}^- T_D} \qquad (12)$$

Substituting Eq.(4) into Eq. (12), then

$$P(x(t) \geq Z_{\min}) = \exp\left[-\frac{\sigma_{\dot{x}} T_D}{2\pi \sigma_x} \exp\left[-\frac{(Z_{\min} - \bar{x})^2}{2\sigma_x^2}\right]\right] \qquad (13)$$

3.2 Calculation and Analysis of Gear Dynamic

Take a typical gear example as the object of this study, the values of the parameters in the differential equation of gear vibration are taken as follows: $\xi_m = 0.05$, $b_m = 0.07$, $\omega_m = 0.75$, $\xi_p = 0.01$, $\xi_g = 0.01$, $b_p = 0$, $b_g = 0$, $e_1 = 0.01$. The gear system under the above parameters of the random vibration system is called 'case one'. The dynamic response of the nonlinear vibration system with random parameters is calculated by the numerical simulation method in Section 2.2, and the dynamical response is shown in Figure 2.

The reliability method which is described in Section 3.1 is used to calculate the reliability of gear vibration response with random parameters: $\mu_x = \bar{x} = 0.15745$, $Z_{\text{threshold}} = 0.05$, $Z_{\max} = \mu_x + Z_{\text{threshold}} = 0.20745$, $Z_{\min} = \mu_x - Z_{\text{threshold}} = 0.10745$. Other calculated parameters are $T_D = 8.3775$, $\sigma_x = 0.014922$, $\sigma_{\dot{x}} = 0.01291$. According to Eq. (6), the reliability of gear dynamic vibration is $R = 0.9916171$.
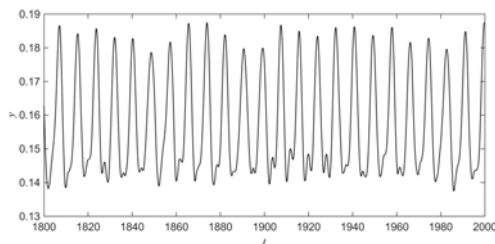

Figure 2: Time history of periodic vibration response for gear system with random parameters (Case one).

When $\omega_m = 1.55$, other parameters of the current nonlinear gear system remain unchanged, the gear system is in a chaotic vibration state. Figure 3 corresponds to the gear system dynamic response time history curve. The reliability method which is described in Section 3.1 is used to calculate the reliability of gear vibration response with random

parameters: $\mu_x = \bar{x} = 0.14856$, $Z_{\text{threshold}} = 0.05$, $Z_{\max} = \mu_x + Z_{\text{threshold}} = 0.19856$, $Z_{\min} = \mu_x - Z_{\text{threshold}} = 0.04977$. Other calculated parameters are $T_D = 4.0537$, $\sigma_x = 0.05651$, $\sigma_{\dot{x}} = 0.01291$. And according to Eq. (6), the reliability of gear dynamic vibration is $R = 0.4638098$.
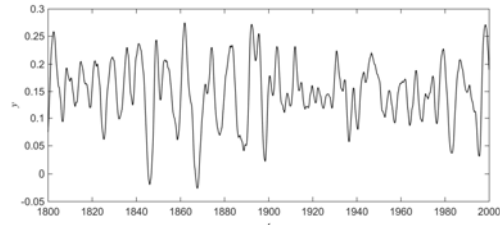

Figure 3: Time history of periodic vibration response for gear system with random parameters (Case two).

Table 1 shows the reliability of different random vibration system response results. By comparison, it is found that the parameter stochastic process has little changes on the vibration reliability of the system for the case one (gear vibration response in a periodic motion). However, for the second case (the vibration response of the gear is chaotic), the vibration reliability rapidly decreases from 0.9417517 to 0.4638098 due to the random process characteristics of the parameters. Results show that chaotic motion system itself is very sensitive to small changes of obove parameters, and the randomness of parameters will lead to the changes of gear dynamic response, so as to cause the individual amplitude of vibration response to be too large. This unpredictable result, which is sensitive to the initial parameters, is in good agreement with chaos theory.

Table 1: Calculation results of each random vibration system

System type	Response type	Reliability calculation results	
		Determined parameters	Random parameters
Case one	Periodic motion	0.9911982	0.9916171
Case two	Chaotic movement	0.9417517	0.4638098

4 CONCLUSIONS

In this paper, the numerical simulation of gear nonlinear vibration system with random parameters are carried out. The calculation formula of the

vibration reliability of gear vibration system with random parameters is deduced and its application range is extended. The comparison of examples shows that the parameter stochastic process has little effect on the vibration reliability of the system when the gear system's response is periodic motion, while the vibration reliability of the system will decrease sharply when the gear system's response is chaotic motion.

This study will provide a reference and theoretical basis for the control and judgment of the nonlinear vibration of gears with random parameters.

ACKNOWLEDGEMENTS

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