Optimization for Urban Rail Transit Network Planning Evaluation Based on Virtual Unit and AHP-DEA Model

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Abstract: Taking the optimization method for urban rail transit network planning as the background, this paper analyses the characteristics of existing evaluation methods, and establishes a comprehensive AHP-DEA evaluation model based on the traditional DEA method. The first stage of the model calculates the weight of criterion level as to the target level by AHP, and the second stage using DEA to calculate the efficiency index of the plan level for the criterion level. Finally, the overall weight of each plan is obtained from the two stages. In addition, the problem of DEA aberration is solved by introducing the virtual unit. Finally, the proposed method is applied and shows good effectiveness.

1 INTRODUCTION

Urban rail transit is a large-investment and long-cycle project, and it is difficult to change once built. Therefore, the decision of network plan is particularly important. At present, the main evaluation methods in related literature can be summarized as two major categories:qualitative and quantitative.

The representative of qualitative method is analytic hierarchy process (AHP)（Saaty TL,1990）, in which the subjective decision of decision-makers playing a decisive role. In the field of urban rail planning, it's crucial for experts to make decisions based on the planning experience of other cities. However, it is difficult for decision-makers to achieve satisfactory decisions when faced with too many decisions. Therefore, the quantitative method is the mainstream choice in present studies, which mainly includes: entropy method(Qian B.Y.and Zhao L.,2017), grey relational analysis(Ren L.,2010), fuzzy hierarchical evaluation(Li J.F.and Wu X.P.,2007), conventional data envelopment analysis (DEA) (Zhang Y.Z., Yan Y.S.，Jiang N.,and Zhang H.W.,2010)etc.

While analyzing the drawback of traditional DEA, this paper puts forward a new method combining the advantage of AHP and DEA，which not only reserves the qualitative evaluation of the indicators, but also conveys the preference of decision-maker. In addition, a virtual DMU is introduced to optimize the solution dilemma where DMUs are effective at the same time, which shows good robustness in solving the evaluation of urban railways network planning problem.

2 IMPROVED DEA MODEL

2.1 The Conventional DEA model

DEA is a well-known method for efficiency measurement based on multiple inputs and multiple outputs which is originated by Charnes, Cooper, and Rhodes (1978). Assuming there are n DMUs with m dimensional input vector and s dimensional output vector, we can define the ith input and output of DMU_j as X_{ij} and Y_{ij}，taking the ratio of output and input as the efficiency index θ_j to seek the best combination of weight values for the decision makers , the initial CCR DEA model for evaluating the efficiency of DMU_0 can be presented as follows(Wei Q.L., 2004)：

$$\begin{cases} max\,\theta_j = \mu^T Y_0 \\ \quad s.t. \\ \omega^T X_j - \mu^T Y_j \geq 0,\ j = 1,2\ldots,n \\ \quad \omega^T X_0 = 1 \\ \quad \omega \geq 0, \mu \geq 0 \end{cases} \tag{1}$$

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The conventional DEA model can identify efficient and inefficient DMUs. Those with efficiency index equal to 1 are called efficient while DMUs with efficiency index are less than 1 are called inefficient. Obviously, inefficient DMUs can be ranked by efficiency index directly, whereas efficient DMU can't achieve that because there are generally more than one efficient DMU，which is called "DEA aberration"（Wu Y.H., Zeng X.Y., Song J.W.,1999）. The more inputs and outputs there are, the more serious the aberration will be.

2.2 The introduction of virtual unit

When it comes to using the CCR DEA model to evacuate urban rail transit network planning, the aberration is prominent as a result of few DMUs along with a large amount of inputs and outputs. Aiming at this problem, this paper introduces an optimal virtual DMU with the minimum input and the maximum output based on the literature (Wang L., Liu C.R.,2010), so that DMUs can be further distinguished. The virtual DMU can be defined as follows:

Assuming that $x_i = \min_{1\le j\le n}\left\{x_{ij}\right\}(i=1,2,...,m)$, $y_r = \max_{1\le j\le n}\left\{y_{rj}\right\}(r=1,2,...,s)$, then the virtual DMU can be presented by $(X_{n+1}、Y_{n+1})$,where $X_{n+1}=(x_1,x_2,...,x_m)^T$ 、 $Y_{n+1}=(y_1,y_2,...,y_s)^T$, the improved DEA model based on virtual DMU can be presented as followed:

$$\begin{cases} max\,\mu^T Y_0 \\ s.t. \\ \omega^T X_j - \mu^T Y_j \ge 0, j=1,2\ldots,n+1 \\ \omega^T X_0 = 1 \\ \omega \ge 0, \mu \ge 0 \end{cases} \quad (2)$$

In the model, the input and output of the original decision unit is replaced by the virtual unit, so that the efficiency index is reduced relative to the virtual unit., therefore the decision units (network plans) can be ranked accordingly.

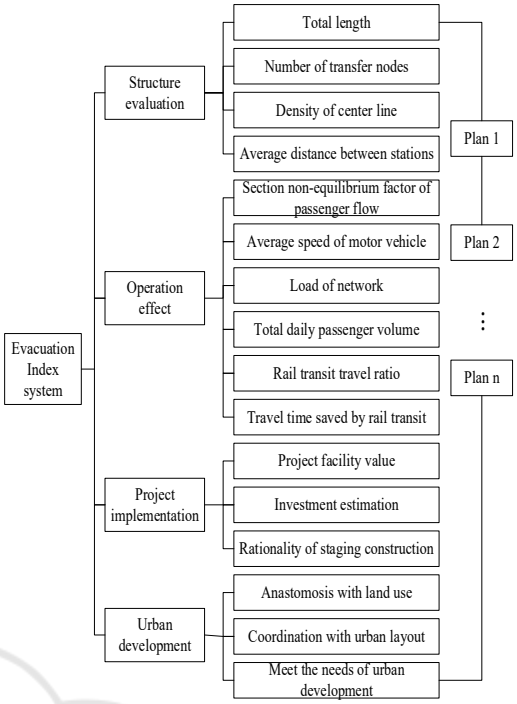

Figure 1: The evaluation index system

2.3 The AHP-DEA Comprehensive Evacuation Model

In the process of urban rail network planning decision, allowing for factors such as land use and urban layout coordination, decision-makers often have subjective preferences, which directly affect the final decision. Allowing for it, the paper introduces AHP to reflect preference of decision-makers and constructs a AHP-DEA comprehensive valuation model based on virtual units. The structure of the model is shown in Figure 2.

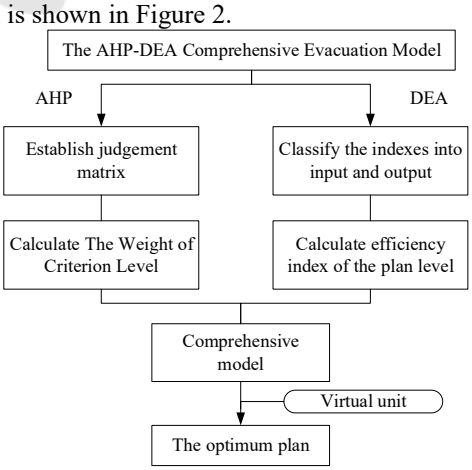

Figure 2: The structure of the AHP-DEA comprehensive evaluation model.

2.4 Calculate Weight of Criterion Level

The first step is establishing the judgment matrix A_{ij} according to the "1-9 scale method", the maximum eigenvalue λ_m and eigenvector W are obtained, where AW=λ_mW, then the n component of W is the weight of the n factors correspondingly.After calculating, consistency check is necessary to ensure the reasonablity of A_{ij}.

2.5 The efficiency index of the plan level to the criterion level

As for criterion i (i=1, 2, 3, 4, representing structural evaluation, project implementation, operation effect and urban development, respectively), classifying the indexes of evaluation system by taking the cost index and the benefit index as input and output respectively, we can define the rth input and output of DMU_j as x_{rj} and y_{rj}, regard each plan as a DMU, and establish DEA model based on virtual unit for each criterion by the method referenced in Section 2 :

$$E_{ij} = \max \sum_{r,j} u_r y_{rj}$$

$$\begin{cases} \sum_{r,j} v_r x_{rj} - \sum_{r,j} \mu_r y_{rj} \geq 0,\ j = 1,2,..n+1 \\ \sum_{r,j} v_r x_{rj} = 1 \\ u_r \geq 0, v_r \geq 0; r = 1,2,\ldots \end{cases} \quad (3)$$

Where E_{ij} represents the evacuation index of the j scheme for criterion i.

2.6 Comprehensive Evacuation

Calculating the overall weight of each plan M_j based on W_i and E_{ij} obtained above:

$$M_j = \sum_{i=1}^{4} e_{ij} w_i \quad (4)$$

3 NUMERICAL EXAMPLES

In this section, we present a numerical example *Urban rapid rail transit network planning in Changsha* taken from related literature(Meng X.D.,2007) to apply the new proposed AHP-DEA model for selecting the most efficient DMU, The data for this example is given in table 1.

The specific steps are as follows:

①Establish the evaluation index system for urban rail transit network planning (as shown in Figure 1);

②Calculate the weight of the criterion level as to total level:

Table 1: Evaluation index of urban rail transit network planning in Changsha.

Type	Index name	Plan 1	Plan 2	Plan 3
Input index	Total length	172.1	163.9	172.2
	Number of transfer nodes	14	10	12
	Section non-equilibrium factor of passenger flow	2.36	2.59	2.65
	Project facility value	6.68	7.83	8.14
	Investment estimation	8.61	7.9	7.84
Output index	Density of center line	31	29	31
	Average station spacing	2.1	2.07	2.24
	Average speed of motor vehicle	80	75	80
	Load of network	3.51	4.06	3.48
	Total daily passenger volume	376.6	364.6	372.2
	Rail transit travel ratio	34.76	36.39	37.49
	Travel time saved by rail transit	11.88	11.81	12.12
	Rationality of staging construction	8.22	7.9	7.47
	Anastomosis with land use	8.61	7.9	7.84
	Coordination with urban layout	6.39	6.42	6.62
	Meet the needs of urban development	8.74	7.7	7.55

The judgement matrix A=$\begin{bmatrix} 1 & 1 & 5 & 3 \\ 1 & 1 & 3 & 4 \\ 1/5 & 1/3 & 1 & 1/2 \\ 1/3 & 1/4 & 2 & 1 \end{bmatrix}$ is constructed and then the maximum eigenvalue λ_m and eigenvector W are obtained: λ_m=4.085 , W=$(\omega_i)^T$ =$(0.6894, 0.6699, 0.1514, 0.2287)^T$ .Where ω_i is the weight of the criterion as to total level correspondingly.

③Calculate the efficiency index of the plan level as to criterion level;

Table 2 is the efficiency index of each plan for each criterion calculated based on the method referenced in Section 3.4:

④Comprehensive evacuation. Table 3 gives the calculation result of overall weights of plan 1、plan 2 and plan 3：

Table 2: Efficiency index of plan level to criterion level.

Evaluation criteria	Plan 1	Plan 2	Plan 3
structure evaluation	0.952	0.935	1.000
operation effect	1.000	0.911	0.891
Project Implementation	1.000	0.902	0.908
urban development	1.000	0.884	0.920

Table 3: The overall weight.

DMU	Plan 1	Plan 2	Plan 3
overall weight	1.709	1.596	1.637

As we see from Table 4：M_1>M_3>M_2, plan 1 is the most efficient DMU ranked as the top position, which is consistent with the result of the literature (Meng X.D.,2007) obtained by improved multi-objective decision making model, as well as the final result of Changsha urban rail transit network planning. It is proved that the AHP-DEA comprehensive evaluation model based on virtual unit proposed in this paper is feasible. And for the decision makers, this model could be further applied in performance evaluation.

4 CONCLUSION

Based on the existing DEA model(CCR), this paper introduces the AHP to reflect the preference of decision-maker in evacuation of urban rail transit network planning, and a comprehensive evacuation AHP-DEA model is proposed for finding the optimum plan. Furthermore, confronted with the problem that the traditional DEA model may appear all effective DMUs, when there are multiple inputs and multiple outputs (especially the number of DMU is far less than the number of indexes), a virtual unit is introduced in order to distinguish DMUs, which provides a good solution to DEA aberration, thus the proposed model is of strong practicability compared with tradition model. However, the current model doesn't consider the select of input and output data in detail, which is an issue in the latest literatures. Further important future research directions would be selecting the more efficient data for the model by additional restraints or developing models to deal with fuzzy data.

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