Makespan Minimization with Sequence-dependent Non-overlapping Setups

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Abstract: This paper deals with a scheduling problem that emerges in the production of water tubes of different sizes that require reconfiguration of the machines. The reconfiguration of the machines leads to the notion of sequence-dependent setup times between tasks. These setups are often performed by a single person who cannot serve more than one machine at the same moment, i.e., the setups must not overlap. Surprisingly, the problem with non-overlapping setups has received only a little attention so far. To solve this problem, we propose an Integer Linear Programming formulation, Constraint Programming models and a hybrid heuristic that leverages the strength of Integer Linear Programming in the shortest Hamiltonian path problem and the efficiency of Constraint Programming at sequencing problems with makespan minimization. The experimental evaluation shows that among the proposed exact approaches, the Constraint Programming is a superior method being able to solve instances with 3 machines and up to 11 tasks on each machine to optimality within a few seconds. The proposed hybrid heuristic attains high-quality solutions for instances with 50 machines and up to 116 tasks on each machine.

1 INTRODUCTION

The problem studied in this paper is inspired by a continuous production of plastic water tubes. In such productions, the factory brings in the material in form of plastic granulate that is being in-house processed. The manufacturer has a stack of orders for manufacturing plastic tubes of various widths and lengths. The production has 13 machines that can produce different tubes in parallel. Different variants of tubes require different settings of the machines. Hence, when switching from one type of tube to another, a machine setter is required to visit the particular machine and make the tool adjustment. The goal is to process all orders as fast as possible.

As the tool adjustment is done by a single machine setter, he or she is likely to be the bottleneck of the production when the orders are not scheduled well. Given the assignment of the orders to the machines, the basic idea is to cluster similar tube variants next to each other, as these require little or no setup time to adjust the tool.

We model the problem as a scheduling problem where the tasks are dedicated to the machines and have sequence-dependent setup times. Each setup occupies an extra resource that is assumed to be unary, hence setups must not overlap in time. The goal is to minimize the makespan of the overall schedule. In this paper, we design an Integer Linear Programming (ILP) model, three variants of Constraint Programming (CP) model, and a heuristic algorithm.

The main contributions of this paper are:

- formal definition of a new problem with non-overlapping setups
- exact approaches based on ILP and CP formalisms
- a very efficient hybrid heuristic yielding optimal or near-optimal schedules

The rest of the paper is organized as follows. We first survey briefly the existing work in the related area. Next, Section 3 gives the formal definition of the problem at hand. In Section 4, we describe an ILP model, while in Section 5 we introduce three

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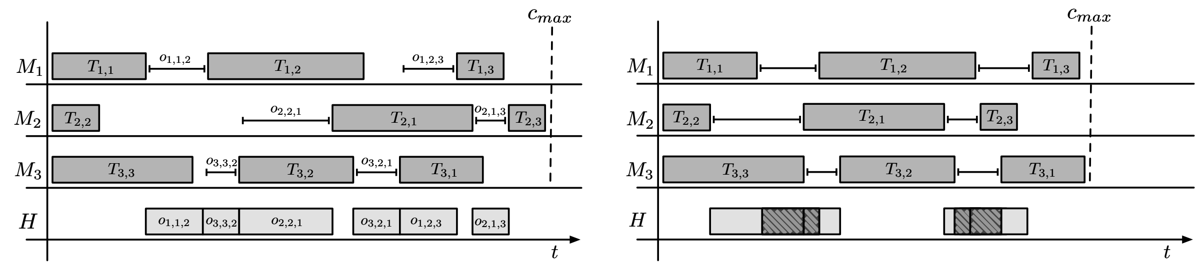


(a) Feasible schedule.

(b) Infeasible schedule, setups are overlapping.

Figure 1: An illustration of a schedule with three machines and three tasks to be processed on each machine.

variants of a CP model, and in Section 6 we propose the heuristic algorithm. Finally, we present computational experiments in Section 7 and draw conclusions in Section 8.

2 RELATED WORK

There is a myriad of papers on scheduling with sequence-dependent setup times or costs (Allahverdi et al., 2008), proposing exact approaches (Lee and Pinedo, 1997) as well as various heuristics (Vallada and Ruiz, 2011). But the research on the problems where the setups require extra resource is scarce.

An unrelated parallel machine problem with machine and job sequence-dependent setup times, studied by (Ruiz and Andrés-Romano, 2011), considers also the non-renewable resources that are assigned to each setup, which affects the amount of time the setup needs and which is also included in the objective function. On the other hand, how many setups may be performed at the same time is disregarded. The authors propose a Mixed Integer Programming formulation along with some static and dynamic dispatching heuristics.

A lotsizing and scheduling problem with a common setup operator is tackled in (Tempelmeier and Buschkühl, 2008). The authors give ILP formulations for what they refer to as a dynamic capacitated multi-item multi-machine one-setup-operator lotsizing problem. Indeed, the setups to be performed by the setup operator are considered to be scheduled such that they do not overlap. However, these setups are not sequence-dependent in the usual sense. The setups are associated to a product whose production is to be commenced right after the setup and thus the setup time, i.e., the processing time of the setup, does not depend on a pair of tasks but only on the succeeding task.

A complex problem that involves machines requiring setups that are to be performed by operators of different capabilities has been addressed in (Chen et al., 2003). The authors modeled the whole problem in the time-indexed formulation and solved it by decomposing the problem into smaller subproblems using Lagrangian Relaxation and solving the subproblems using dynamic programming. A feasible solution is then composed of the solutions to the subproblems by heuristics, and, if impossible, the Lagrangian multipliers are updated using surrogate subgradient method as in (Zhao et al., 1999). The down side of this approach is that the time-indexed formulation yields a model of pseudo-polynomial size. This is not suitable for our problem as it poses large processing and setup times.

To the best of our knowledge, this is the first paper that efficiently solves the scheduling problem with dedicated machines with sequence-dependent non-overlapping setups.

3 PROBLEM STATEMENT

Informally speaking, the problem tackled in this paper consists of a set of machines and a set of independent non-preemptive tasks, each of which is dedicated to one particular machine where it will be processed. Also, there are sequence-dependent setup times on each machine. In addition, these setups are to be performed by a human operator who is referred to as a machine setter. Such a machine setter cannot perform two or more setups at the same time. It follows that the setups on all the machines must not overlap in time. Examples of a feasible and an infeasible schedule with 3 machines can be seen in Fig. 1. Even though the schedule (Fig. 1b) on the machines contains setup times, such schedule is infeasible since it would require overlaps in the schedule for the machine setter.

The aim is to find a schedule that minimizes the completion time of the latest task. It is clear that the latest task is on some machine and not in the schedule of a machine setter since the completion time of the last setup is followed by at least one task on a machine.

3.1 Formal Definition

Let $M = \{M_1, ..., M_m\}$ be a set of machines and for each $M_i \in M$, let $T^{(i)} = \{T_{i,1}, ..., T_{i,n_i}\}$ be a set of tasks that are to be processed on machine M_i, and let $T = \bigcup_{M_i \in M} T^{(i)} = \{T_{1,1}, ..., T_{m,n_m}\}$ denote the set of all tasks. Each task $T_{i,j} \in T$ is specified by its processing time $p_{i,j} \in \mathbb{N}$. Let $s_{i,j} \in \mathbb{N}_0$ and $C_{i,j} \in \mathbb{N}$ be start time and completion time, respectively, of task $T_{i,j} \in T$, which are to be found. All tasks are non-preemptive, hence, $s_{i,j} + p_{i,j} = C_{i,j}$ must hold.

Each machine $M_i \in M$ performs one task at a time. Moreover, the setup times between two consecutive tasks processed on machine $M_i \in M$ are given in matrix $O^{(i)} \in \mathbb{N}^{n_i \times n_i}$. That is, $o_{i,j,j'} = (O^{(i)})_{j,j'}$ determines the minimal time distance between the start time of task $T_{i,j'}$ and the completion time of task $T_{i,j}$ if task $T_{i,j'}$ is to be processed on machine M_i right after task $T_{i,j}$, i.e., $s_{i,j'} - C_{i,j} \geq o_{i,j,j'}$ must hold.

Let $H = \{h_1, \ldots, h_\ell\}$, where $\ell = \sum_{M_i \in M} n_i - 1$, be a set of setups that are to be performed by the machine setter. Each $h_k \in H$ corresponds to the setup of a pair of tasks that are scheduled to be processed in a row on some machine. Thus, function $st : H \to M \times T \times T$ is to be found. Also, $s_k \in \mathbb{N}_0$ and $C_k \in \mathbb{N}$ are start time and completion time of setup $h_k \in H$, which are to be found. Assuming $h_k \in H$ corresponds to the setup between tasks $T_{i,j} \in T$ and $T_{i,j'} \in T$, i.e., $st(h_k) = (M_i, T_{i,j}, T_{i,j'})$, it must hold that $s_k + o_{i,j,j'} = C_k$, also $C_{i,j} \leq s_k$, and $C_k \leq s_{i,j'}$. Finally, since the machine setter may perform at most one task at any time, it must hold that, for each $h_k, h_{k'} \in H, k \neq k'$, either $C_k \leq s_{k'}$ or $C_{k'} \leq s_k$.

The objective is to find such a schedule that minimizes the makespan, i.e., the latest completion time of any task:

$$\min \max_{T_{i,j} \in T} C_{i,j} \tag{1}$$

It is easy to see that such problem is strongly $\mathcal{NP}$-hard even for the case of one machine, i.e., $m = 1$, which can be shown by the reduction from the shortest Hamiltonian path problem.

In the following sections, we propose two exact approaches.

4 INTEGER LINEAR PROGRAMMING MODEL

The proposed formulation models the problem with two parts. The first part handles scheduling of tasks on the machines using efficient *rank-based model* (Lasserre and Queyranne, 1992). This approach uses binary variables $x_{i,j,q}$ to encode whether task $T_{i,j} \in T^{(i)}$ is scheduled on q-th position in the permutation on machine $M_i \in M$. Another variable is $\tau_{i,q}$ denoting the start time of a task that is scheduled on q-th position in the permutation on machine $M_i \in M$.

The second part of the model resolves the question, in which order and when the setups are performed by a machine setter. There, we need to schedule all setups H, where the setup time π_k of the setup $h_k \in H$ is given by the corresponding pair of tasks on the machine.

Let us denote the set of all natural numbers up to n as $[n] = \{1, \ldots, n\}$. We define the following function $\phi : H \to M \times [\max_{M_i \in M} n_i]$ (e.g., $\phi(h_k) = (M_i, q)$), that maps $h_k \in H$ to setups between the tasks scheduled at positions q and $q+1$ on machine $M_i \in M$. Since the time of such setup is a variable (i.e., it depends on the pair of consecutive tasks on M_i), *rank-based model* would not be linear. Therefore, we use the relative-order (also known as disjunctive) model (Applegate and Cook, 1991; Balas, 1968) that admits processing time given as a variable. Its disadvantage over the rank-based model is that it introduces a *big M* constant in the constraints, whereas the rank-based model does not. See Fig. 2 for meaning of the variables.

The full model is stated as:

$$\min \; C_{max} \tag{2}$$

s.t.

$$C_{max} \geq \tau_{i,n_i} + \sum_{T_{i,j} \in T^{(i)}} p_{i,j} \cdot x_{i,j,n_i} \; \forall M_i \in M \tag{3}$$

$$\sum_{q \in [n_i]} x_{i,j,q} = 1 \quad \forall M_i \in M, \forall T_{i,j} \in T^{(i)} \tag{4}$$

$$\sum_{T_{i,j} \in T^{(i)}} x_{i,j,q} = 1 \quad \forall M_i \in M, \forall q \in [n_i] \tag{5}$$

$$s_k + \pi_k \leq s_l + \mathcal{M} \cdot (1 - z_{k,l}) \quad \forall h_l, h_k \in H : l < k \tag{6}$$

$$s_l + \pi_l \leq s_k + \mathcal{M} \cdot z_{k,l} \; \forall h_l, h_k \in H : l < k \tag{7}$$

$$\pi_k \geq o_{i,j,j'} \cdot (x_{i,j,q} + x_{i,j',q+1} - 1) \quad \forall h_k \in H : \phi(h_k) = (M_i, q), \forall T_{i,j}, T_{i,j'} \in T^{(i)} \tag{8}$$

$$s_k + \pi_k \leq \tau_{i,q+1} \quad \forall h_k \in H : \phi(h_k) = (M_i, q) \tag{9}$$

$$s_k \geq \tau_{i,q} + \sum_{T_{i,j} \in T^{(i)}} p_{i,j} \cdot x_{i,j,q} \tag{10}$$

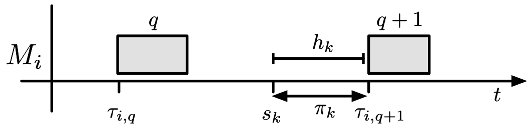


Figure 2: Meaning of the variables in the model.

$$\forall h_k \in H : \phi(h_k) = (M_i, q)$$

where

$$C_{max} \in \mathbb{R}_0^+ \tag{11}$$

$$\tau_{i,q} \in \mathbb{R}_0^+ \quad \forall M_i \in M, \forall q \in [n_i] \tag{12}$$

$$s_k, \pi_k \in \mathbb{R}_0^+ \quad \forall h_k \in H \tag{13}$$

$$x_{i,j,q} \in \{0,1\} \tag{14}$$

$$\forall M_i \in M, \forall T_{i,j} \in T^{(i)}, \forall q \in [n_i]$$

$$z_{k,l} \in \{0,1\} \quad \forall h_k, h_l \in H : l < k \tag{15}$$

The constraint (3) computes makespan of the schedule while constraints (4)–(5) states that each task occupies exactly one position in the permutation and that each position is occupied by exactly one task. Constraints (6) and (7) guarantee that setups do not overlap. $\mathcal{M}$ is a constant that can be set as $|H| \cdot \max_i O^{(i)}$. Constraint (8) sets processing time π_k of the setup $h_k \in H$ to $o_{i,j,j'}$ if task $T_{i,j'}$ is scheduled on machine M_i right after task $T_{i,j}$. Constraints (9) and (10) are used to avoid conflicts on machines. The constraint (9) states that a task cannot start earlier than its preceding setup finishes. Similarly, the constraint (10) states that a setup is scheduled after the corresponding task on the machine finishes.

4.1 Formulation for a Single Machine

The problem with a single machine ($M_i \in M$) reduces to the shortest Hamiltonian path in the graph defined by setup time matrix $O^{(i)}$. To solve this problem, we transform it to the Traveling Salesperson Problem by introducing a dummy task $T_{i,0} \in T^{(i)\prime} = T^{(i)} \cup \{T_{i,0}\}$ that has zero setup times with all other tasks, i.e., $o_{i,0,j} = o_{i,j,0} = 0$, $\forall T_{i,j} \in T^{(i)\prime}$. Then, we use a well-known sub-tour elimination (Applegate et al., 2011; Pferschy and Staněk, 2017) ILP model to solve it:

$$\min \sum_{T_{i,j} \in T^{(i)\prime}} \sum_{T_{i,j'} \in T^{(i)\prime}} o_{i,j,j'} \cdot y_{j,j'} + \sum_{T_{i,j} \in T^{(i)\prime}} p_{i,j} \tag{16}$$

s.t.

$$\sum_{T_{i,j} \in T^{(i)\prime}} y_{j,j'} = 1 \quad \forall T_{i,j'} \in T^{(i)\prime} \tag{17}$$

$$\sum_{T_{i,j'} \in T^{(i)\prime}} y_{j,j'} = 1 \quad \forall T_{i,j} \in T^{(i)\prime} \tag{18}$$

$$\sum_{T_{i,j}, T_{i,j'} \in S} y_{j,j'} \le |S| - 1 \quad \forall S \subset T^{(i)\prime} \tag{19}$$

where

$$y_{j,j'} \in \{0,1\} \quad \forall T_{i,j}, T_{i,j'} \in T^{(i)\prime} \tag{20}$$

The variable $y_{j,j'}$ indicates whether task $T_{i,j}$ is immediately followed by task $T_{i,j'}$. We solve the model in a lazy way, i.e., without constraints (19), that are lazily generated during the solution by a depth-first search algorithm. Note that the machine setter does not need to be modeled for the single machine problem.

4.2 Additional Improvements

We use the following improvements of the model that have a positive effect on the solver performance.

1. **Warm Starts.** The solver is supplied with an initial solution. It solves a relaxed problem, where it relaxes on the condition that setups do not overlap. Such solution is obtained by solving the shortest Hamiltonian path problem given by setup time matrix $O^{(i)}$ independently for each machine $M_i \in M$, as described in Section 4.1. Since such solution might be infeasible for the original problem, we transform it in a polynomial time into a feasible one. It is done in the following way. For each setup among all machines, we set the start time of k-th setup on machine M_i, $i \ge 2$, to the completion time of k-th setup on machine M_{i-1}. For the setups on machine M_1, the start time of $(k+1)$-th setup is set to the completion time of k-th setup on machine M_m.
2. **Lower Bounds.** We supply a lower bound on C_{max} variable given as the maximum of all best proven lower bounds of model (16)–(20) among all machines $M_i \in M$ (see Section 4.1).
3. **Pruning of Variables.** We can reduce the number of variables in the model due to the structure of the problem. We fix values of some of the $z_{k,l}$ variables according to the following rule. Let $h_k, h_l \in H$ such that $\phi(h_k) = (M_i, q)$ and $\phi(h_l) = (M_i, v)$ for any $M_i \in M$. Then, $q < v \Rightarrow z_{k,l} = 1$ holds in some optimal solution. Note that the rule holds only for setups following from the same machine.

   The rule states that the relative order of setups on the same machine is determined by the natural ordering of task positions on that machine. See for example setups $o_{1,1,2}$ and $o_{1,2,3}$ in Fig. 1. Since these setups follow from the same machine, their relative order is already predetermined by positions of the respective tasks. We note that the presolve of the solver was not able to deduce these rules on its own.

5 CONSTRAINT PROGRAMMING MODELS

Another way how the problem at hand can be tackled is to use the modeling approach based on the Constraint Programming (CP) formalism, where special global constraints modeling unary (disjunctive) resources and efficient filtering algorithms are used (Vilím et al., 2005). These concepts work with *interval variables* whose start time and completion time are denoted by predicates $StartOf$ and $EndOf$, and the difference between the completion time and the start time of the interval variable can be set using predicate $LengthOf$.

The CP models are constructed as follows. We introduce interval variables $I_{i,j}$ for each $T_{i,j} \in T$, and the lengths of these interval variables are set to the corresponding processing times:

$$LengthOf(I_{i,j}) = p_{i,j} \tag{21}$$

The sequence is resolved using the $NoOverlap$ constraint. The $NoOverlap(I)$ constraint on a set I of interval variables states that it constitutes a chain of non-overlapping interval variables, any interval variable in the chain being constrained to be completed before the start of the next interval variable in the chain. In addition, the $NoOverlap(I,\ O^{(i)})$ constraint is given a so-called *transition distance* matrix $O^{(i)}$, which expresses a minimal delay that must elapse between two successive interval variables. More precisely, if $I_{i,j}, I_{i,j'} \in I$, then $(O^{(i)})_{j,j'}$ gives a minimal allowed time difference between $StartOf(I_{j'})$ and $EndOf(I_j)$. Hence, the following constraint is imposed, $\forall M_i \in M$:

$$NoOverlap\Big(\bigcup_{T_{i,j} \in T^{(i)}} \{I_{i,j}\}\ ,\ O^{(i)} \Big) \tag{22}$$

The objective function is to minimize the makespan:

$$\min \max_{T_{i,j} \in T} EndOf(I_{i,j}) \tag{23}$$

This model would already solve the problem if the setups were not required to be non-overlapping. In what follows we describe three ways how the non-overlapping setups are resolved. Constraints (21)–(23) are part of each of the following model.

5.1 CP1: with Implications

Let us introduce $I^{st}_{i,j}$ for each $T_{i,j} \in T$ representing the setup after task $T_{i,j}$. There is $\sum_{M_i \in M} n_i$ such variables. To ensure that the setups do not overlap in time is enforced through the following constraint:

$$NoOverlap\Big(\bigcup_{T_{i,j} \in T} \{I^{st}_{i,j}\} \Big) \tag{24}$$

Notice that this constraint is only one and it is over all the interval variables representing setups on all machines. This $NoOverlap$ constraint does not need any transition distance matrix as the default values 0 are desired.

Since it is not known a priori which task will be following task $T_{i,j}$, the quadratic number of implications determining the precedences and lengths of the setups must be imposed. For this purpose, the predicate $Next$ is used. $Next(I)$ equals the interval variable that is to be processed in the chain right after interval variable I. Thus, the following constraints are added, $\forall M_i \in M, \forall T_{i,j}, T_{i,j'} \in T^{(i)}, j \neq j'$:

$$Next(I_{i,j}) = I_{i,j'} \Rightarrow EndOf(I_{i,j}) \leq StartOf(I^{st}_{i,j}) \tag{25}$$

$$Next(I_{i,j}) = I_{i,j'} \Rightarrow EndOf(I^{st}_{i,j}) \leq StartOf(I_{i,j'}) \tag{26}$$

$$Next(I_{i,j}) = I_{i,j'} \Rightarrow LengthOf(I^{st}_{i,j}) = o_{i,j,j'} \tag{27}$$

Note that the special value when an interval variable is the last one in the chain is used to turn the last setup on a machine into a dummy one.

5.2 CP2: with Element Constraints

We did not find a way how to avoid the quadratic number of implications for setting the precedences, but at least setting the lengths of the setups can be substituted by the element constraint, which might be beneficial as global constraints are usually more efficient. More precisely, this model contains also constraints (24), (25), and (26), but constraint (27) is substituted as follows.

Assume the construct $Element(Array, k)$ returns the k-th element of $Array$, $(O^{(i)})_j$ is the j-th row of matrix $O^{(i)}$, and $IndexOfNext(I_{i,j})$ returns the index of the interval variable that is to be processed right after $I_{i,j}$. Then the following constraint is added, for each $I^{st}_{i,j}$:

$$LengthOf(I^{st}_{i,j}) = Element\Big((O^{(i)})_j,\ IndexOfNext(I_{i,j})\Big) \tag{28}$$

5.3 CP3: with Optional Interval Variables

In this model, we use the concept of *optional interval variables* (Laborie et al., 2009). An optional interval variable can be set to be *present* or *absent*. The predicate $PresenceOf$ is used to determine whether or not the interval variable is present in the resulting schedule. Whenever an optional interval variable is absent, all the constraints that are associated with that optional interval variable are implicitly satisfied and predicates $StartOf$, $EndOf$, and $LengthOf$ are set to 0.

Hence, we introduce optional interval variable $I^{opt}_{i,j,j'}$ for each pair of distinct tasks on the same machine, i.e., $\forall M_i \in M, \forall T_{i,j}, T_{i,j'} \in T^{(i)}, j \neq j'$. There are $\sum_{M_i \in M} n_i(n_i - 1)$ such variables. The lengths of these interval variables are set to corresponding setup times:

$$LengthOf(I^{opt}_{i,j,j'}) = o_{i,j,j'} \tag{29}$$

To ensure that the machine setter does not perform more than one task at the same time, the following constraint is added:

$$NoOverlap\Big(\bigcup_{\substack{T_{i,j}, T_{i,j'} \in T \\ j \neq j'}} \{I^{opt}_{i,j,j'}\}\Big) \tag{30}$$

In this case, to ensure that the setups are indeed processed in between two consecutive tasks, we use the constraint $EndBeforeStart(I_1, I_2)$, which ensures that interval variable I_1 is completed before interval variable I_2 can start, but if either of the interval variables is absent, the constraint is implicitly satisfied. Thus, the following constraints are added, $\forall I^{opt}_{i,j,j'}$:

$$EndBeforeStart(I_{i,j}, I^{opt}_{i,j,j'}) \tag{31}$$

$$EndBeforeStart(I^{opt}_{i,j,j'}, I_{i,j'}) \tag{32}$$

Finally, in order to ensure the correct presence of optional interval variables, the predicate $PresenceOf$ is used. Thus, the following constraint is imposed, $\forall I^{opt}_{i,j,j'}$:

$$PresenceOf(I^{opt}_{i,j,j'}) \Leftrightarrow Next(I_{i,j}) = I_{i,j'} \tag{33}$$

5.4 Additional Improvements

We use the following improvements:

1. **Search Phases.** Automatic search in the solver is well tuned-up for most types of problems, leveraging the newest knowledge pertaining to variable selection and value ordering heuristics. In our case, however, preliminary results showed that the solver struggles to find any feasible solution already for small instances. It is clear that it is easy to find some feasible solution, e.g., by setting an arbitrary order of tasks on machines and then shifting the tasks to the right such that the setups do not overlap. To make the solver find always some feasible solution at a blow, we set the search phases such that the sequences on machines are resolved first, and then the sequences of setups for the machine setter are resolved. This is included in all the CP models described.

2. **Warm Starts.** Similarly to improvement (1) in Section 4.2, we boost the performance by providing the solver with a starting point. We do this only for CP3 as the preliminary numerical experiments showed a slight superiority of CP3.

   More precisely, we first find an optimal sequence of tasks minimizing makespan on each machine separately, as described in Section 4.1, and then we set those interval variables $I^{opt}_{i,j,j'}$ to be present if $T_{i,j'}$ is sequenced directly after $T_{i,j}$ on machine M_i. This is all that we set as the starting point. Notice that unlike in Section 4.2, we do not calculate the complete solution but we let the solver do it. The solver then quickly completes the assignment of all the variables such that it gets a solution of reasonably good objective value.

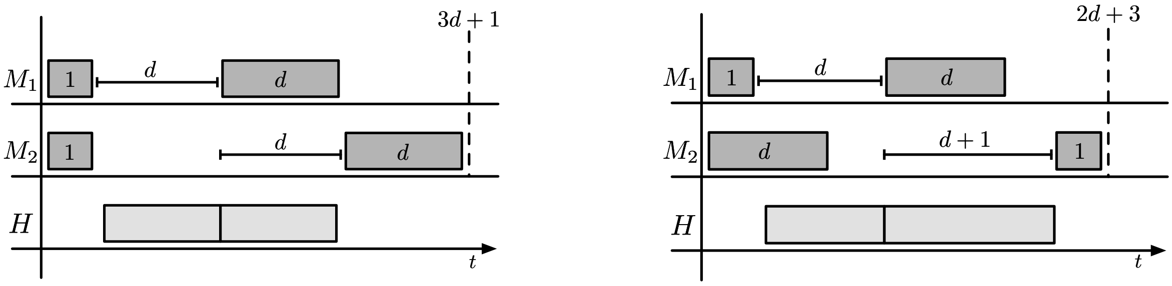
   Note that the optimal sequences on machines are solved using ILP so it can be seen as a hybrid approach. This model with warm starts is in what follows referred to as *CP3ws*.

6 HEURISTIC APPROACH

We propose an approach that guides the solver quickly towards solutions of very good quality but cannot guarantee optimality of what is found. There are two main phases of this approach. In the first phase, the model is decomposed such that its subproblems are solved optimally or near-optimally and then the solutions of the subproblems are put together so as to make a correct solution of the whole problem. In the second phase, the solution found is locally improved by repeatedly adjusting the solution in promising areas. More details follow.

6.1 Decomposition Phase

The idea of the model decomposition is as follows. First, again, we find an optimal sequence of tasks minimizing makespan on each machine separately, as described in Section 4.1. Second, given these sequences on each machine, the setups to be performed are known, hence, the lengths of the setups are fixed as well as the precedence constraints with respect to



(a) A problem instance where optimal sequences on machines lead to a sub-optimal solution.

(b) Sub-optimal sequence on one machine yields a globally optimal solution.

Figure 3: Solving the problem greedily for each machine separately can lead to arbitrarily bad solutions. The numbers depict the processing times of the tasks and setups.

the tasks on machines. Thus, all that needs to be resolved is the order of setups.

The pseudocode is given in Algorithm 1. It takes one machine at a time and finds an optimal sequence for it while minimizing makespan. The time limit for the computation of one sequence on a machine is given in such a way that there is a proportional remaining time limit for the rest of the algorithm. OPTIMALSEQ(i, *TimeLimit*) returns the best sequence it finds on machine $M_i \in M$ in the given *TimeLimit*. The *TimeLimit* is computed using *RemainingTime*(), which is the time limit for the entire run of the algorithm minus the time that already elapsed from the beginning of the run of the algorithm. In the end, the solution is found using the knowledge of the sequences on each machine $M_i \in M$.

Algorithm 1: Solving the decomposed model.

function SOLVEDECOMPOSED
  for each $M_i \in M$ **do**
    TimeLimit $\leftarrow$ *RemainingTime*()$/(m-i+2)$
    $Seq_i \leftarrow$ OPTIMALSEQ(i, *TimeLimit*)
  end for
  Return SOLVE(*Seq*, *RemainingTime*())
end function

Clearly, this decomposition may lead to a schedule arbitrarily far from the optimum. Consider a problem depicted in Fig. 3. It consists of two machines, M_1 and M_2, and two tasks on each machine, with processing times $p_{1,1} = p_{2,1} = 1, p_{1,2} = p_{2,2} = d$, where d is any constant greater than 2, and with setup times $o_{1,1,2} = o_{2,1,2} = d, o_{1,2,1} = o_{2,2,1} = d+1$. Then, optimal sequence on each machine yields a solution of makespan $3d+1$, whereas choosing sub-optimal sequence on either of the machines gives optimal objective value $2d+3$.

6.2 Improving Phase

Once we have some solution to the problem, the idea of the heuristic is to improve it applying the techniques known as local search (Hentenryck and Michel, 2009) and large neighborhood search (Pisinger and Ropke, 2010).

It is clear that in order to improve the solution, something needs to be changed on the *critical path*, which is such a sequence of setups and tasks on machines that the completion time of the last task equals the makespan and that none of these tasks and setups can be shifted to the left without violating resource constraints (see an example in Fig. 4). Hence, we find the critical path first.

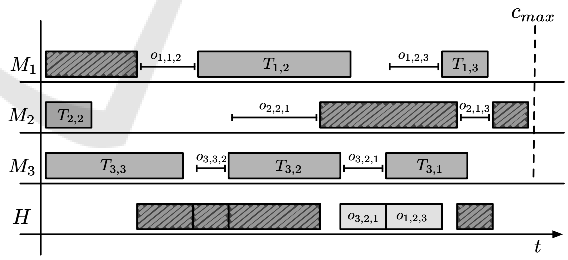


Figure 4: An illustration of the critical path depicted by dashed rectangles.

The most promising place to be changed on the critical path could be the longest setup. Hence, we find the longest setup on the critical path, then we prohibit the two consecutive tasks corresponding to the setup from being processed in a row again and re-optimize the sequence on the machine in question. Two tasks are precluded from following one another by setting the corresponding setup time to infinite value. Also, we add extra constraint restricting the makespan to be less than the incumbent best objective value found. The makespan on one machine being equal to or greater than the incumbent best objective value found cannot lead to a better solution.

After a new sequence is found, the solution to the whole problem is again re-optimized subject to the new sequence. The algorithm continues this way until the sequence re-optimization returns infeasible, which happens due to the extra constraint restricting the makespan. It means that the solution quality deteriorated too much and it is unlikely to find a better solution locally at this state. Thus, the algorithm reverts to the initial solution obtained from the decomposed model, restores the original setup times matrices, and tries to prohibit another setup time on the critical path. For this purpose, the list of *nogoods* to be tried is computed once from the first critical path, which is just a list of setups on the critical path sorted in non-increasing order of their lengths. The whole iterative process is repeated until the total time limit is exceeded or all the nogoods are tried.

The entire heuristic algorithm is hereafter referred to as LOFAS (Local Optimization for Avoided Setup). The pseudocode is given in Algorithm 2.

Preliminary experiments confirmed the well-known facts that ILP using lazy approach, as described in Section 4.1, is very efficient for searching an optimal sequence on one resource, and CP is more efficient for minimizing makespan when the lengths of interval variables and the precedences are fixed. That is why the best results are achieved using ILP from Section 4.1 for OPTIMALSEQ(i, $TimeLimit$) and CP for SOLVE(Seq, $RemainingTime()$).

Algorithm 2: Local Optimization for Avoided Setup.

```
function LOFAS
    S^init ← SOLVEDECOMPOSED
    S^best ← S^init
    P_crit ← critical path in S^init
    nogoods ← {h_k ∈ H ∩ P_crit}
    sort nogoods in non-increasing order of lengths
    for each h_k ∈ nogoods do
        h_k' ← h_k
        while true do
            (M_i, T_i,j, T_i,j') ← st(h_k')
            o_i,j,j' ← ∞
            add: max_{T_i,j ∈ T^(i)} C_i,j < ObjVal(S^best)
            TimeLimit ← RemainingTime()/2
            Seq_i ← OPTIMALSEQ(i, TimeLimit)
            if Seq_i is infeasible then
                Revert to S^init
                Restore original O^(i), ∀M_i ∈ M
                break
            end if
            S^new ← SOLVE(Seq, RemainingTime())
            if ObjVal(S^best) > ObjVal(S^new) then
                S^best ← S^new
            end if
            if RemainingTime() ≤ 0 then
                return S^best
            end if
            P_crit ← critical path in S^new
            h_k' ← longest setup ∈ {h_k ∈ H ∩ P_crit}
        end while
    end for
    return S^best
end function
```

7 EXPERIMENTAL RESULTS

For the implementation of the constraint programming approaches, we used the IBM CP Optimizer version 12.8 (Laborie et al., 2018). The only parameter that we adjusted is `Workers`, which is the number of threads the solver can use and which we set to 1.

For the integer programming approach, we used Gurobi solver version 8 (Gurobi, 2018). The parameters that we adjust are `Threads`, which we set to 1, and `MIPFocus`, which we set to 1 in order to make the solver focus more on finding solutions of better quality rather than proving optimality. We note that parameters tuning with Gurobi Tuning Tool did not produce better values over the baseline ones.

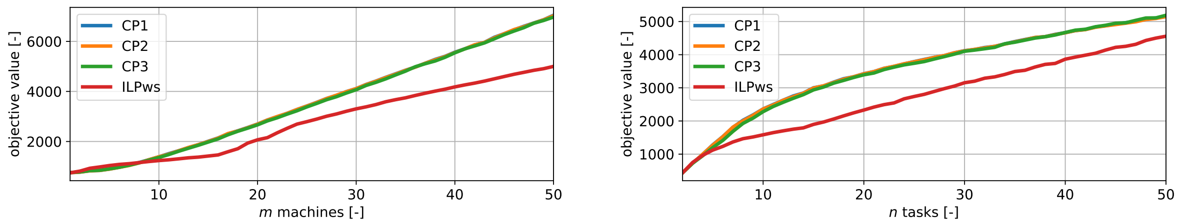
The experiments were run on a Dell PC with an Intel® Core™ i7-4610M processor running at 3.00 GHz with 16 GB of RAM. We used a time limit of 60 seconds per problem instance.

7.1 Problem Instances

We evaluated the approaches on randomly generated instances of various sizes with the number of machines m ranging from 1 to 50 and the number of tasks on each machine $n_i = n$, $\forall M_i \in M$, ranging from 2 to 50. Thus, we generated $50 \times 49 = 2450$ instances in total. Processing times of all the tasks and setup times are chosen uniformly at random from the interval $[1, 50]$. Instances are publicly available at `https://github.com/CTU-IIG/NonOverlappingSetupsScheduling`.

7.2 Results

Figure 5a shows the dependence of the best objective value found by the exact approaches within the 60s time limit on the number of machines, averaged over the various number of tasks. Analogically, Fig. 5b



(a) Mean objective value for different number of machines m.

(b) Mean objective value for different number of tasks n.

Figure 5: Comparison of exact models.

shows the dependence of the best objective value on the number of tasks, averaged over the varying number of machines.

The results show that the performances of CP models are almost equal (the graphs almost amalgamate). We note that CP3 is the best but the advantage is negligible. Hence, we will not further distinguish between them and we will use the minimum of all three CP models that will be referred to as *CPmin*.

On the other hand, the inclusion of the ILP approach with the warm starts (henceforth referred to as *ILPws*) in this comparison is inappropriate in that the CP models do not get any warm start. When the ILP approach model did not get the initial solution as a warm start, it was not able to find any solution even for very small instances (i.e., 2 machines and 8 tasks). In fact, the objective value found by the ILPws is often the objective value of the greedy initial solution given as the warm start (i.e., Section 4.2).

Further, we compare the best objective value found by the heuristic algorithm LOFAS from Section 6 against CPmin. Figure 6a shows the dependence on the number of machines, while Fig. 6b shows the dependence on the number of tasks. Note that we omit the results of CP3ws (CP3 model with warm starts) in Fig. 6 as the results were almost the same as those of LOFAS and the curves amalgamated.

To further compare LOFAS to CP3ws, we generated instances of size up to 120 tasks on each machine ($119 \times 50 = 5950$ instances in total). The optimality of a solution was proved by CP3ws in total for 75 instances, and out of these 75 instances, LOFAS rendered worse solution only for 6 instances, thus giving an optimal solution in 92 % of instances. The smallest instance for which CP3ws did not find any solution consisted of 50 machines and 72 tasks on each machine, whereas the smallest instance for which LOFAS did not find any solution contained 117 tasks on each machine. Out of these 5950 instances, CP3ws did not find any solution for 1018 instances, while LOFAS did not find any solution only for 53 instances. From instances that were solved by both algorithms, CP3ws yielded a better solution than LOFAS only for 641 instances, whereas LOFAS gave a better solution than CP3ws for 2329 instances. Finally, the biggest difference in the objective values found was 3.74 % in favor of LOFAS, but only 2.45 % in favor of CP3ws.

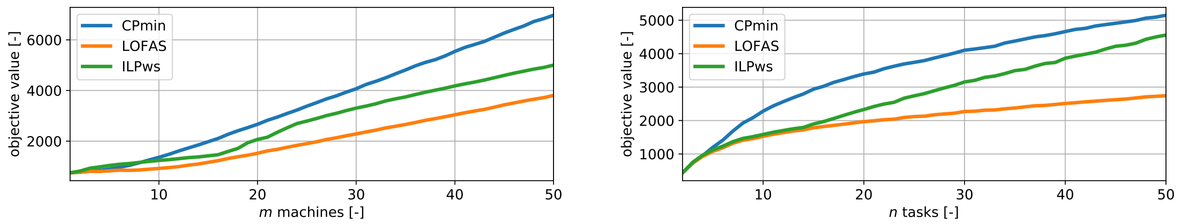
The reason why LOFAS did not find any solution to the biggest instances was that the time limit was exceeded during the decomposition phase, i.e., during seeking an optimal sequence for a machine. Hence, the performance of LOFAS can be significantly improved if a better TSP solver, e.g., Concorde (Applegate et al., 2011), would be used instead of the model from Section 4.1. This is not the case for CP3ws, which did not manage to combine the solutions to the subproblems together already for smaller instances.

Note that the comparison of CP3ws to ILPws, which is shown in Fig. 7, is legit, as they both get a warm start in a certain sense, and confirms lower performance of the ILP approach.

To obtain better insight into the performance of the proposed methods, we compared the resulting distributions of achieved objectives from each method. For each method, we took results for all instances and ordered them in a non-decreasing way with respect to achieved objective value and plotted them. The results are displayed in Fig. 8. It can be seen that the proposed heuristic is able to find the same or better solutions in nearly all cases. Moreover, one can notice a spike at around 65 % of instances for ILPws. This is caused by the fact that for some instances, the ILP solver was not able to improve upon the initial warm start solution in the given time limit and these instances thus contribute to the distribution with higher objective values.

7.3 Discussion

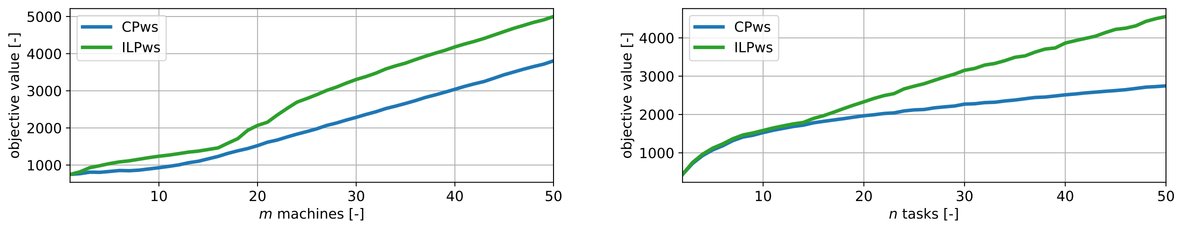
We have seen that performances of CP models are almost equal with CP3 being the best but its advantage is negligible. Further, the experiments have shown that ILP without a warm start cannot find a feasi-



(a) Mean objective value for different number of machines m.

(b) Mean objective value for different number of tasks n.

Figure 6: Comparison of exact models and the heuristic algorithm.



(a) Mean objective value for different number of machines m.

(b) Mean objective value for different number of tasks n.

Figure 7: Comparison of exact models with warm starts.

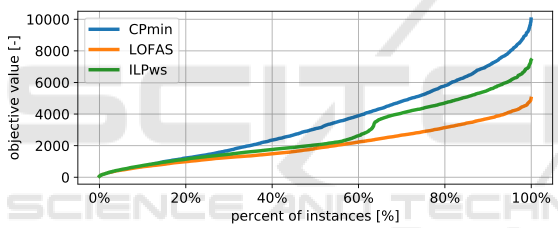


Figure 8: Objective distributions of different methods.

ble solution for instances with $n \geq 8$ tasks reliably, whereas with warm starts it was significantly better than the best CP model without a warm start. The quality of the solutions from CP with warm starts is much better than ILP with warm starts, as can be seen in Fig. 7. As expected, the heuristic algorithm LOFAS produced the best solutions among all compared methods, although only slightly better than CP3 model with warm starts. From smaller instances it can be seen that LOFAS achieves objective values quite close to optimal ones.

8 CONCLUSIONS

In this paper, we tackled the problem of scheduling on dedicated machines with sequence-dependent non-overlapping setups. We suggested an ILP model, three variants of a CP model and a heuristic algorithm. The extensive experimental evaluation showed that all exact models themselves are yielding solutions far from optima within the given time limit of 60 seconds, which proved them inappropriate mainly for larger instances. However, the proposed heuristic algorithm that combines ILP and CP yields high-quality solutions in very short computation time. The gist is that we leveraged the strength of ILP in the shortest Hamiltonian path problem and the efficiency of CP at sequencing problems with makespan minimization.

For future work, a more complex problem will be considered. The main limitation of the model proposed in this paper is that tasks are assumed to be already assigned to machines. In practice, it may happen that each task can be processed on some subset of machines. Also, instead of non-overlapping setups for one machine setter, there may be more machine setters that must be then treated as a resource with limited capacity. In addition, this capacity may vary in time (e.g., to avoid night shifts). Another key feature of many real-life production problems is the presence of release times and deadlines or precedence constraints. For such a problem, finding an initial feasible schedule will be already a non-trivial problem and the solution approach from Section 4.1 for the case of a single machine does not work anymore.

Next, before a machine setter can perform a setup, it may require moving to another machine and preparing some tools, which may lead to a concept of setups over setups. It would require a setup times matrix of size $O(|T|^4)$, which does not seem plausible. However, if we settle for the setup times over setups to be determined by the pair of machines where the two consecutive setups are performed, which yields a setup times matrix of size only $O(m^2)$, it could bring the problem closer to real-life applications.

ACKNOWLEDGEMENTS

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