Exhaustive Testing of Trader-agents in Realistically Dynamic Continuous Double Auction Markets: AA Does Not Dominate

Dave Cliff
Department of Computer Science
University of Bristol, Bristol BS8 1UB, U.K.

Keywords: Financial Markets, Automated Trading, Computational Finance, Agent-based Computational Economics.

Abstract: We analyse results from over 3.4million detailed market-trading simulation sessions which collectively confirm an unexpected result: in markets with dynamically varying supply and demand, the best-performing automated adaptive auction-market trading-agent currently known in the AI/Agents literature, i.e. Vytelingum's *Adaptive-Aggressive* (AA) strategy, can be routinely out-performed by simpler trading strategies. AA is the most recent in a series of AI trading-agent strategies proposed by various researchers over the past twenty years: research papers contributing major steps in this evolution of strategies have been published at *IJCAI*, in the *Artificial Intelligence* journal, and at *AAMAS*. The innovative step taken here is to brute-force exhaustively evaluate AA in market environments that are in various ways more realistic, closer to real-world financial markets, than the simple constrained abstract experimental evaluations routinely used in the prior academic AI/Agents research literature. We conclude that AA can indeed appear dominant when tested only against other AI-based trading agents in the highly simplified market scenarios that have become the methodological norm in the trading-agents academic research literature, but much of that success seems to be because AA was designed with exactly those simplified experimental markets in mind. As soon as we put AA in scenarios closer to real-world markets, modify it to fit those markets accordingly, and exhaustively test it against simpler trading agents, AA's dominance simply disappears.

1 INTRODUCTION

Automated *algorithmic trading* systems are a big business. In most major financial markets around the world, jobs previously done by highly-paid human traders are now routinely done by machines, autonomous adaptive computational systems that can process vast amounts of data and that can act and react at speeds that no human is physically capable of matching. Commonly referred to as "algo traders" or "robot traders", such automated systems running in any one major investment bank might be responsible for order-flows of $100Bn or more per working week. When it comes to sub-second financial-market trading, we *Homo Sapiens* are simply made from the wrong hardware: in the global financial markets, the "rise of the robots" has been underway for the past 10 years or more. And, inside that industry, everybody knows the robots won (see e.g. Rodgers, 2016).

The AI and Autonomous Agents research community should be claiming this as a victory, a major demonstration of success. If the annual compensation (salary and bonuses) paid to someone in a knowledge-intensive job is even a half-way reasonable indication of the intelligence required to do that job, then the fact that traders previously paid very high levels of compensation have now been replaced by machines costing only a tiny fraction of a trader's salary is surely a sign that, in the domain of the financial markets, the widespread deployment of artificially intelligent "robot trader" autonomous agents is a major success story for AI/Agents research. Such a claim can be justified by reference to the key published literature on adaptive automated trading. Although a few significant publications contributing to the development of robot-trading systems came from academic economists, the landmark papers largely appeared in AI and autonomous-agent publication venues such as the *International Joint Conference on Artificial Intelligence* (IJCAI), the *International Conference on Autonomous Agents and Multi-Agent Systems* (AAMAS), the *Artificial Intelligence* journal (AIJ), and in previous ICAART papers: Section 2 reviews in more detail eight major publications in the development of this field.
The review in Section 2 is important, because there we

Cliff, D.
Exhaustive Testing of Trader-agents in Realistically Dynamic Continuous Double Auction Markets: AA Does Not Dominate.
DOI: 10.5220/0007382802240236
In *Proceedings of the 11th International Conference on Agents and Artificial Intelligence (ICAART 2019)*, pages 224-236
ISBN: 978-989-758-350-6

trace the way in which the methodology of initial experiments published in 1962 by a young economist, Vernon Smith (who 40 years later would be awarded the Nobel Prize for his empirical research work) have since come to be fixed, or fixated upon, in the AI/agents literature. Motivated by what it seems fair to assume was a well-intentioned desire to show each set of the latest results in the context of what had gone before, papers that followed Smith's replicated much or all of his 1962 experiment design and analysis. And this, it seems, may have led down a dead end.

The papers reviewed here are a sequence of steps, each building on the earlier work, that lead to the most recent step: Vytelingum's 2006 PhD thesis which described a trading algorithm called *Adaptive Aggressive* (AA) that, in an AIJ paper (Vytelingum *et al.*, 2007), and in subsequent *ICAART* and *IJCAI* papers (De Luca and Cliff, 2012a, 2012b), was demonstrated to be the best-performing public-domain trading strategy. That is, AA was shown to perform better, for a specific definition of "better", than all other notable strategies published in the literature up to that time. (It is possible that better strategies exist and are being used to profitably trade in real financial markets, but for obvious reasons any such strategies would be closely-guarded commercial secrets: we can only talk with any authority about those strategies known in the public domain).

In this paper we demonstrate that the trading capabilities of AA are, when faced with realistic market dynamics, in fact really rather limited. While AA does very well in the type of minimally-simple abstract market experiments that had become the norm for evaluating and comparing trading-agent algorithms, we argue here that this success is due to AA having been seemingly (and perhaps subconsciously) designed specifically to address features of those abstract experimental markets, features that are absent or much more complicated in real-world markets. It is as if, somewhere along the line, people collectively lost sight of the fact that the ultimate test of any automated trading system claimed to be relevant to the real-world financial markets is simply stated: *how much money can it actually make?* The results presented here demonstrate that, when operating in a realistically dynamic market, AA routinely makes less money than simpler strategies.

This paper reports on simulation experiments involving more than 3 million independent market sessions where AA and various other trading strategies interact and compete for limited profits, in a market with a Limit Order Book (LOB), the core data structure found in real-world financial markets, explained further in Section 2.2. We use the free open-source *BSE* LOB-market simulator (BSE, 2012; Cliff, 2018) available from GitHub since 2012. Using a well-established public-domain market simulator makes it easier for other researchers to check, replicate, and extend our methods and results.

After the review of past work in Section 2, in Section 3 we briefly discuss issues arising, modifications that need to be made, to adapt AA from its original design (which is extremely well-suited to minimal abstract market experiments) over to working in the much more realistic environments presented by a contemporary market simulator such as BSE. To distinguish between the original AA, which does not operate in realistic markets, and the version modified and extended to work in BSE, we here refer to the latter as *Modified AA* (MAA). We describe our methods in Section 4, and our results in Section 5.

We start in Section 5.1 by replicating the spirit of prior work, comparing MAA's performance to other trading agents in a BSE market that is deliberately constrained to match the simplifying assumptions and constraints of earlier experimental work. After that, we explore the effects of removing those simplifying assumptions and constraints: we report the resultant changes in the relative performance of MAA and those other trading strategies active in the market alongside it. We find that when the market's underlying supply and demand schedules are stationary (i.e., are largely fixed for the course of an experiment) or suffer intermittent step-change "price shocks", MAA does as well as all previous publications lead us to expect. However, when we introduce dynamic variation into the supply and demand schedules over the course of individual experiments, such that the market's equilibrium price is continuously varying, we then find that MAA's performance degenerates badly. Section 5.2 then presents results from experiments where fluctuations in the equilibrium price are driven by a price-history taken from a real-world asset, for a variety of asset-classes. We find that these real-world dynamics lead MAA to always do worse than simpler strategies.

Section 6 then discusses these results and concludes that the success of MAA seems to be due in large extent to "methodological over-fitting", i.e. to being embedded in a research methodology so set on repeating the same style of experiments (admirably so, because replication is fundamental to validation), that it lost sight of what real-world phenomena those experiments were intended to be abstract models of. MAA does very well in the abstract experimental scenarios, but it does so well in comparison to its terrible performance in more realistic scenarios that it is hard to avoid the conclusion that AA was (perhaps subconsciously) designed *specifically for* those abstract models without much consideration of its performance in more realistic environments like actual financial markets. Unfortunately for MAA, practitioners in real financial markets are not at all forgiving of mismatches between models and reality. The ultimate message of this paper is that we should all be testing our systems in as realistic environments as we have reasonable access to. Free access to public-domain open-source market simulators such as BSE, and to alternatives such as *OpEx* (De Luca, 2015) or *ExPo* (Stotter *et al.*, 2013), coupled with cheaply available cloud computing, now makes this kind of study much easier for others to replicate.

2 BACKGROUND

The 2002 Nobel Prize in Economics was awarded to Vernon Smith, in recognition of Smith's work in establishing and thereafter growing the field of *Experimental Economics* (abbreviated hereafter to "ExpEcon"). Smith showed that the microeconomic behaviour of human traders interacting within the rules of some specified market, known technically as an *auction mechanism*, could be studied empirically, under controlled and repeatable laboratory conditions, rather than in the noisy messy confusing circumstances of real-world markets. The minimal laboratory studies could act as useful proxies for studying real-world markets of any type, but one particular auction mechanism has received the majority of attention: the *Continuous Double Auction* (CDA), in which any buyer can announce a bid-price at any time and any seller can announce an offer-price at any time, and in which at any time any trader in the market can accept an offer or bid from a counterparty, and thereby engage in a transaction. The CDA is the basis of major financial markets worldwide.

Smith's initial set of experiments were run in the late 1950's, and the results and associated discussion were presented in his first paper on ExpEcon, published in the highly prestigious *Journal of Political Economy* (JPE) in 1962. It seems plausible to speculate that when his JPE paper was published, Smith had no idea that it would mark the start of a line of research that would eventually result in him being appointed as a Nobel laureate. And it seems even less likely that he would have foreseen the extent to which the experimental methods laid out in that 1962 paper would subsequently come to dominate the methodology of researchers working to build adaptive autonomous trading agents by combining tools and techniques from artificial intelligence (AI), machine learning (ML), agent-based modelling (ABM), and agent-based computational economics (ACE). Although not a goal stated at the outset, this strand of AI/ML/ABM/ACE research converged toward a common aim: specifying an artificial agent, an autonomous adaptive trading strategy, that could automatically tune its behaviour to different market environments, and that could reliably beat all other known automated trading strategies, thereby taking the crown of being the current best trading strategy known in the public domain, i.e., the "dominant strategy". Over the past 20 years the dominant strategy crown has passed from one algorithm to another. Here, we demonstrate that the current holder of the title, Vytelingum's (2006) *AA* strategy, does not perform nearly so well as was previously believed from success in small numbers of simple trials.

Given that humans who are reliably good at trading are generally thought of as being "intelligent" in some reasonable sense of the word, the aim to develop ever more sophisticated artificial trading systems is clearly within the scope of AI research, although some very important early ideas came from the economics literature: a comprehensive review of relevant early research was presented by Cliff (1997). Below in Section 2.1 we first briefly introduce eight key publications leading to the development of AA; then describe key aspects of ExpEcon market models in Section 2.2; and then discuss each of the eight key publications in more detail in Section 2.3.

2.1 A Brief History of Trading Agents

Our story starts with Smith's 1962 JPE paper. The next major step comes 30 years later, with a surprising result published in the JPE by Gode and Sunder (1993): this popularised a minimally simple automated trading algorithm now commonly referred to as *ZIC*. A few years later two closely related research papers were published independently and at roughly the same time, each written without knowledge of the other: the first was a Hewlett-Packard Labs technical report by Cliff (1997) describing the adaptive AI/ML trading-agent strategy known as the *ZIP* algorithm; the second summarised the PhD thesis work of Gjerstad, in a paper co-authored with his PhD advisor (Gjerstad and Dickhaut 1998), describing an adaptive trading algorithm now widely known simply as *GD*.
After graduating his PhD, Gjerstand worked at IBM's

TJ Watson Labs where he helped set up an ExpEcon laboratory that his IBM colleagues used in a study that generated world-wide media coverage when the results were published by Das *et al.* at IJCAI-2001. This paper presented results from studies exploring the behaviour of human traders interacting with GD and ZIP robot traders, in a CDA with a Limit Order Book (LOB: explained in more detail in Section 2.2, below), and demonstrated that both GD and ZIP reliably outperformed human traders. Neither GD nor ZIP had been designed to work with the LOB, so the IBM team modified both strategies for their study. A follow-on 2001 paper by Tesauro and Das (two co-authors of the IBM IJCAI paper) described a more extensively *Modified GD* (MGD) strategy, and later Tesauro and Bredin (2002) described the *GD eXtended* (GDX) strategy. Both MGD and GDX were each claimed to be the strongest-known public-domain trading strategies at the times of their publication.

Subsequently, Vytelingum's 2006 thesis introduced the *Adaptive Aggressive* (AA) strategy which, in an AIJ paper (Vytelingum *et al.*, 2007), and in later *ICAART* and *IJCAI* papers (De Luca and Cliff 2012a, 2012b), was shown to be dominant over ZIP, GDX, and human traders. Thus far then, AA holds the title.

However Vach (2015) recently presented results from experiments with the *OpEx* market simulator (De Luca, 2015), in which AA, GDX, and ZIP were set to compete against one another, and in which the dominance of AA is questioned: Vach's results indicate that whether AA dominates or not can be dependent on the ratio of AA:GDX:ZIP in the experiment: for some ratios, Vach found AA to dominate; for other ratios, it was GDX. Vach studied only a relatively small sample from the space of possible ratios, but his results prompted the work reported here, in which we exhaustively sample a wide range of differing ratios of four trading strategies (AA, ZIC, ZIP, and the minimally simple SHVR strategy described in Section 2.2), doing a brute-force search for situations in which AA is outperformed by the other strategies. The combinatorics of such a search are quite explosive, and in Section 5 we report on results from over 3.4 million individual simulations of market sessions. Our findings indicate that Vach's observation was correct: AA's dominance depends on how many other AA traders are in the market; and, in aggregate, AA is routinely outperformed by ZIP and by SHVR.

2.2 On Laboratory Models of Markets

Smith's early experiments were laboratory models of so called *open-outcry trading pits*, a common sight in any real financial exchange before the arrival of electronic trader-terminals in the 1970s. In a trading pit, human traders huddle together and shout out their bids and offers, and also announce their willingness to accept a counterparty's most recent shout. It's a chaotic scene, now largely consigned to the history books. In the closing quarter of the 20th Century, traders moved *en masse* to interacting with each other instead via electronic means: traders "shouted" their offer or bids or acceptances by typing orders on keyboards and then sending those orders to a central server that would display an aggregate summary of all orders currently "shouted" onto the market. That aggregate summary is very often in the form of a *Limit Order Book* or LOB: the LOB shows a summary of all bids and offers currently live in the market. At its simplest, the LOB is a table of numbers, divided into the *bid side* and the *ask side* (also known as the *offer side*). Both sides of the LOB show the best price at the top, with less good prices arranged below in numeric order of price: for the bid side this means the highest-priced bid at the top with the remaining bid prices displayed in descending order below; and for the ask side the lowest-priced offer is at the top, with the remaining offers arranged in ascending order below. The arithmetic mean of the best bid and best ask prices is known as the *mid-price*, and their difference is the *spread.* For each side of the LOB, at each price on the LOB, the quantity available on that side at that price is also indicated, but with no indication of who the relevant orders came from: in this sense the LOB serves not only to aggregate all currently live orders, but also to anonymize them.

Traders in LOB-based markets can usually cancel existing orders to delete them from the LOB. In a common simple implementation of a LOB, traders can accept the current best bid or best offer by issuing a quote that *crosses the spread*: i.e., by issuing an order that, if added to the LOB, would result in the best bid being at a higher price than the best ask. Rather than be added to the LOB, if a bid order crosses the spread then it is matched with the best offer on the ask side (known as *lifting the ask*), whereas an ask that crosses the spread is matched with the best bid (*hitting the bid*); and in either case a transaction then occurs between the trader that had posted the best price on the relevant side of the LOB, and the trader that crossed the spread. The price of the resulting transaction is whatever price was hit or lifted from the top of the LOB.

Smith's earliest experiments pre-dated the arrival of electronic trading in real financial markets, and so they can be thought of as laboratory models of open-outcry trading pits. Even though the much later work by Gode and Sunder, Cliff, Gjerstad and Dickhaut, and Vytelingum all came long after the introduction of electronic LOBs in real markets, these academic studies all stuck with Smith's original methodology, of modelling open-outcry markets (often by essentially operating a LOB with the depth fixed at 1, so the *only* information available to traders is the current best, or most recent, bid and ask prices).

Nevertheless, the studies by IBM researchers (Das *et al.*, 2001; Tesauro and Das, 2001; Tesauro and Bredin 2012), and also the replication and confirmation of AA results by De Luca and Cliff (2011a, 2011b) and by Stotter *et al.* (2013), all used LOB-based market simulators. The IBM simulator *Magenta* seems to have been proprietary to IBM; developed at TJ Watson Labs and not available for third-party use. But De Luca made an open-source release of his *OpEx* simulator (De Luca, 2015) which was subsequently used by Vach (2015) in the studies that prompted our work reported here. Also of relevance here is the *ExPo* simulator described by Stotter *et al.* (2014): in the work by De Luca, by Vach, and by Stotter *et al.*, Vytelingum's original AA needed modifications to make it work in a LOB-based market environment: this is discussed further in Section 3.

In the work reported here we used neither OpEx nor ExPo, but instead BSE (BSE, 2012; Cliff, 2018) which is another open-source ExpEcon market simulator, initially developed as a teaching aid but subsequently used as a platform for research (see, e.g. le Calvez and Cliff, 2018). BSE has the advantage of being relatively lightweight (a single Python script of c.2500 lines) and hence readily deployable over large numbers of virtual machines in the cloud. BSE maintains a dynamically updated LOB and also publishes a *tape*, a time-ordered record of all orders that have been executed. It comes with pre-defined versions of ZIC and ZIP, and also some additionally minimally-simple non-adaptive trading strategies that can be used for benchmarking against other more complex strategies added by the user. One of these, the *Shaver* strategy (referred to in BSE by the "ticker symbol" SHVR) simply reads the best prices on the LOB and, if it is able to do so without risking a loss-making deal then it issues an order that improves the current best bid or best ask by one penny/cent.

2.3 Eight Key Papers, One Methodology

2.3.1 Smith 1962

Smith's 1962 JPE paper is widely regarded as the first published study in ExpEcon. In it he reported on experiments in which groups of human subjects were randomly assigned to be either *buyers* or *sellers*. Buyers were given a supply of artificial money, and sellers were given one or more identical items, of no intrinsic value, to sell. Each trader in the market was assigned a private valuation, a secret *limit price*: for a buyer this was the price above which he or she should not pay when purchasing an item; for a seller this was the price below which he or she should not sell an item. These limit-price assignments model the client orders executed by sales traders in real financial markets; we'll refer to them just as *assignments* in the rest of this paper. After the allocation of assignments to all subjects, they then interacted via an open-outcry CDA while Smith and his assistants made notes on the sequence of events that unfolded during the experiment: typically, buyers would gradually increase their bid-prices, and sellers would gradually lower their offer-prices (also known as ask-prices) until transactions started to occur. Eventually, typically after 5 or 10 minutes, the experimental market reached a position in which no more trades could take place, which marked the end of a *trading period* or "trading day" in the experiment; any one experiment typically ran for n=5-10 periods, with all the traders being resupplied with money and items-for-sale at the start of each trading period. The sequence of n contiguous trading periods (or an equivalently long single-period experiment with continuous replenishment, as discussed in Section 5.1) is referred to here as one *market session*. Smith could induce specific supply and demand curves in these experimental markets by appropriate choices of the various limit-prices he assigned to the traders. As any high-school student of microeconomics knows, the market's theoretical *equilibrium price* (denoted hereafter by P_0) is given by the point where the supply curve and the demand curve intersect. Smith found that, in these laboratory CDA markets populated with only remarkably small groups of human traders, transaction prices could reliably and rapidly converge on the theoretical P_0 value despite the fact that each human trader was acting purely out of self-interest and knew only the limit price that he or she had been assigned. Smith's analysis of his results focused on a statistic that he referred to as α, the root mean square (RMS) deviation of actual transaction prices from the P_0 value over the course of an experiment. In his early experiments, P_0 was fixed for the duration of any one experiment; in later work Smith explored the ability of the market to respond to "price shocks" where, in an experiment of N trading days, on a specific day $S<N$ the allocation of limit prices would be changed, altering P_0 from the value that had been in place over trading periods $1, 2, \ldots, S$, to a different value of P_0 that would then remain constant for the rest of the experiment, i.e. in trading periods $S+1, S+2, \ldots, N$. For brevity, in the rest of this paper Smith's style of experiments will be referred to as *S'62* experiments.

2.3.2 ZIC: Gode and Sunder 1993

Gode and Sunder's 1993 JPE paper used the S'62 methodology, albeit with the CDA markets being electronic (a move Smith himself had made in his experiments many years earlier), so each trader was sat at a personal terminal, a computer screen and keyboard, from which they received all information about the market and via which they announced their orders, their bids or offers, to the rest of the traders in the experiment. Gode and Sunder first conducted a set of experiments in which all the traders were human, to establish baseline statistics. Then, all the human traders were replaced with automated trading systems, absolute-zero minimally-simple algo traders which Gode and Sunder referred to as *Zero Intelligence* (ZI) traders. Gode and Sunder studied markets populated with two type of ZI trader: ZI-Unconstrained (ZIU), which simply generated random prices for their bids or offers, regardless of whether those prices would lead to profitable transactions or to losses; and ZI-Constrained (ZIC), which also generated random order prices but were constrained by their private limit prices to never announce prices that would lead them to loss-making deals. Gode and Sunder used fixed supply and demand schedules in each experiment, i.e. there were no price-shocks in their experiments.

Not surprisingly, the market dynamics of ZIU traders were nothing more than noise. But the surprising result in Gode and Sunder's paper was the revelation that a commonly used metric of market price dynamics known as *allocative efficiency* (AE, hereafter) was essentially indistinguishable between the human markets and the ZIC markets. Because AE had previously been seen as a marker of the degree to which the traders in a market were behaving intelligently, the fact that ZIC traders scored AE values largely the same as humans was a shock. Gode and Sunder proposed that a different metric should instead be used as a marker of the intelligence of traders in the market. This metric was *profit dispersion* (PD, hereafter) which measures the difference between the profit each trader accrued in an experiment, compared to the profit that would be expected for that trader if every transaction in the market had taken place at the market's theoretical equilibrium price P_0: humans typically showed very low values of PD (which is assumed to be good) while ZIC traders did not. On this basis, Gode and Sunder argued that PD should be used in preference to AE.

2.3.3 Zip: Cliff 1997

Taking direct inspiration from both Smith's work and from the ZI paper by Gode and Sunder, Cliff (1997) developed a ZI trading strategy that used simple machine-learning techniques to continuously adapt the randomly-generated prices quoted by the traders: this strategy, known as ZI-Plus (ZIP) was demonstrated to show human-like market dynamics in experiments with flat supply and/or demand curves: Cliff also showed theoretical analyses and empirical results which demonstrated that transaction prices in markets populated only by ZIC traders would not converge to the theoretical equilibrium price when the supply and/or demand curves are flat (or, in the language of microeconomics, "perfectly elastic"). ExpEcon studies in which the supply and/or demand curve was flat had previously been reported by Smith and others, but Gode and Sunder had not explored the response of their ZIC traders to this style of market. Cliff's work involved no human traders: all the focus was on markets populated entirely by autonomous agents, by ZIP traders. In total Cliff (1997) reported on fewer than 1,000 simulated market sessions. The focus on *homogenous* markets can fairly be interpreted as continuing the tradition established by Gode and Sunder (who studied markets homogeneously populated with either human, ZIU, or ZIC traders) and by Smith (who studied all-human markets). In all other regards Cliff continued the S'62 tradition: key metrics were Smith's α, AE, and PD.

2.3.4 GD: Gjerstad and Dickhaut 1997

Gjerstad's PhD studies of price formation in CDA markets (Gjerstad and Dickhaut, 1998) also involved creating an algorithm that could trade profitably by adapting its behavior over time, in response to market events. In contrast to the ZI work, Gjerstad's trading algorithm uses frequentist statistics, gradually constructing and refining a *belief function* that estimates the likelihood for a bid or offer to be accepted in the market at any particular time, mapping from price of the order to its probability of success. Gjerstad did not explicitly name his strategy, but it has since become known as the GD strategy. In all other regards, as with Cliff (1997) and Gode and Sunder (1993), Gjerstad's work was firmly in the S'62 tradition: homogenous markets of GD traders interacting in a CDA, buying and selling single items, with the metrics being Smith's α, AE, and PD.

2.3.5 MGD: IBM 2001

In their landmark 2001 IJCAI paper, IBM researchers Das, Hanson, Kephart, and Tesauro studied the performance of GD and ZIP in a series of ExpEcon market experiments where, for the first time ever in the same market, some of the traders were robots while others were human (recall that the earlier work of Smith, of Gode and Sunder, of Cliff, and of Gjerstad and Dickhaut had all studied homogeneous markets: either all-human or all-robot). Das *et al.* used a LOB-based market simulator called *Magenta,* developed by Gjerstad, and ran a total of six experiments, six market sessions, in which humans and robots interacted and where there were three shock-changes to P_0, i.e. four phases in any one experiment, each phase with a different P_0 value that was held static over that phase. The surprising result in this paper was that robot trading strategies could consistently outperform human traders, by significant margins: a result that attracted worldwide media attention. Both GD and ZIP outperformed human traders, and in the six experiments reported by Das *et al.* the results from the two robot strategies are so similar as to not obviously be statistically significant. A subsequent paper by IBM's Tesauro and Das (2001), reported on additional studies in which a *Modified GD* (MGD) strategy was exhibited what the authors described in the abstract of their paper as *"…the strongest known performance of any published bidding strategy*".

2.3.6 GDX: Tesauro and Bredin 2002

Extensions to MGD were reported by IBM researchers Tesauro and Bredin (2002) at AAMAS 2002. This described extensions to MGD, using dynamic programming methods: this version was named GDX and its performance was evaluated when competing in heterogenous markets with ZIP and other strategies. Tesauro and Bredin reported that GDX outperformed the other strategies and claimed in the abstract of their paper that GDX *"...may offer the best performance of any published CDA bidding strategy*."

2.3.7 AA: Vytelingum 2006

Vytelingum developed AA and documented it in full in his PhD thesis (2006) and in a major paper in the *AIJ* (Vytelingum *et al.,* 2008). The internal mechanisms of AA are described in greater detail in Section 3 of this paper. Although Vytelingum's work came a few years after the IBM publications reviewed in Sections 2.3.5 and 2.3.6, the discussion within Vytelingum's publications is phrased very much in terms of the S'62 methodology: the P_0 value in his AA experiments was either fixed for the duration of each market session, or was subjected to a single "price shock" partway through the session (as described in Section 2.3.1); and again the primary metrics studied are Smith's α, AE, and PD. Vytelingum presented results from heterogeneous market experiments where AA, GDX, and ZIP traders were in competition, and the published results indicated that AA outperformed both GDX and ZIP by small margins. In total, results from c.25,000 market sessions are presented in (Vytelingum *et al.,* 2008).

2.3.8 AA Dominates: De Luca and Cliff 2011

As part of the research leading to his 2015 PhD thesis, De Luca used his LOB-based *OpEx* market simulator system (De Luca, 2012) to study the performance of AA in heterogeneous market experiments where some of the traders were AA, some were other robot strategies such as ZIP, and some were human traders sat at terminals interacting with the other traders (human and robot) in the market via the OpEx GUI, in the style introduced by the IBM team in their IJCAI 2001 paper. De Luca and Cliff (2011a) had previously published results from comparing GDX and AA in OpEx, at ICAART-2011, and the first results from AA in human-agent studies were then published in a 2011 IJCAI paper (De Luca and Cliff, 2011b), in which AA was demonstrated to dominate not only humans but also GDX and ZIP. For consistency with what was by then a well-established methodology, in De Luca's experiments the P_0 value was static for sustained periods with occasional "shock" step-changes to different values. Continuing the tradition established by the IBM authors, the abstract of (De Luca and Cliff 2011b) claimed supremacy for AA: "*We… demonstrate that AA's performance against human traders is superior to that of ZIP, GD, and GDX. We therefore claim that… AA may offer the best performance of any published bidding strategy*".
And, until the publication of Vach (2015), that claim appeared to be plausibly true.

3 MAA: MODIFIED AA

Taking the AA algorithm and attempting to run it in a LOB-based market reveals the extent to which AA seems designed to fit very well in the Smith'62 style of experiments with periodic replenishment, and is less well suited to a continuously varying market dynamic. In brief, AA's internal mechanisms revolve around three questions that each AA trader attempts to answer: (1) What is my best estimate of the current equilibrium price P_0? (2) What is my best estimate of the current volatility of transaction prices around P_0? And (3) is the limit price on my current assignment intramarginal (i.e., could be sold/bought at P_0 and still make a profit) or extramarginal? For its estimate of P_0, the original AA trader computes a moving average of recent transaction prices. For its volatility estimate, it computes Smith's α metric, taking the difference between recent transaction prices and the trader's current estimate of P_0 (i.e., ignoring any trend in P_0, which is safe to do if, as in the S'62 experiments, P_0 changes rarely or never). Deciding on whether the current assignment is intra/extra marginal is done by comparing its limit price to its P_0 estimate.

In MAA, our modified implementation of AA, these questions can instead each be answered by reference to information that is routinely available from an exchange: the LOB and the exchange's "tape" (the record of timestamped transactions). P_0 can be better estimated by using the volume-weighted mid-price at the top of the book (known as the *microprice*): this is a better metric because it can be sensitive to shifts in the P_0 value *before* any transactions go through that reflect the shift. Volatility can be estimated by reference not to only the current estimate of P_0 but also to BSE's tape data: a time-series of transaction-price values correlated with a time series of microprice values is better to use in situations where the P_0 value is continuously changing: for each transaction on the tape, the microprice at the time of that transaction (or immediately before) is the better reference value for calculating Smith's α. Extra-/intra-marginality is still decided by reference to the trader's P_0 estimate, but in MAA that estimate can come from the microprice.

Previous authors have also needed to adapt AA for LOB-based markets: De Luca (2011a, 2011b, 2015) and Vach (2015) each used AA in the *OpEx* simulator, and Stotter *et al.* (2013) used AA in the *ExPo* simulator. However, the modified AA proposed here is novel insofar as prior authors don't report using the exchange's tape data or the microprice.

There is a tension between modifiying AA in an attempt to better fit it to a LOB-based market, and making claims about AA's poor performance in those markets: the more heavily AA is modified, the more one is open to accusations that the modifications themselves are the cause of the poor performance, rather than that poor performance being a reflection of the original AA being badly-suited to LOB markets. For that reason, in this paper, we keep AA very close to the original, using only the microprice modification in generating the results presented here.

4 EVALUATION METHODS

Having modified AA to run in the more realistic CDA market scenarios provided by BSE, we evaluated its performance, measured as average profitability per trader, when tested against other trading strategies under a variety of supply and demand schedules, in markets of varying population sizes; and, for any one population size, testing across an exhaustive sequence of strategy-ratios (described in Section 4.1). In the experiments reported here, we chose to test AA against three other strategies: Gode and Sunder's (1993) ZIC (see Section 2.3.2); Cliff's (1997) ZIP (Section 2.3.3); and the BSE built-in strategy SHVR (Section 2.2). ZIC serves as a lower-limit non-adaptive baseline strategy, albeit one that ignores all information available on the LOB; SHVR as a minimally simple non-adaptive strategy that does actually use LOB data; and ZIP, also pre-coded into BSE, was argued by Vytelingum to be outperformed by AA; and was argued by Tesauro and Bredin to be outperformed by GDX: so if AA cannot do better than ZIP in a specific type of experiment then it presumably also cannot do better than GDX.

For brevity, the only metric that we discuss here for any given strategy, for any one trial or for aggregate results of multiple trials, is the average profit per trader (APPT) calculated across all traders playing that particular strategy, and the associated stddev.

4.1 Varying Trader-Strategy Ratios

The results published by Vach (2015) demonstrated that the measured performance of AA in a heterogeneous market (i.e. a market populated by trading agents with a variety of trading strategies) could be heavily dependent on the ratios of the various strategies active in the market. To control for this, in the experiments reported here we evaluate the performance of trading strategies by calculating summary statistics that aggregate over a large number of trials for any given ratio of the various trading strategies in the market: let T represent the number of trials we perform for any one ratio of trading strategies, and let S represent the number of different strategies we are testing in an experiment. We systematically and exhaustively vary the ratios of the different trading strategies in the market for a given total number of traders in the market, which we refer to as the population size P, which in turn is determined by the number $NEqR$ of traders running each strategy when the ratio is equal across all strategies, such that $P=2{\cdot}S{\cdot}NEqR$. This is best illustrated with an example: in Section 5 we report on experiments with MAA, SHVR, ZIC, and ZIP, so S=4. Then let R denote the ratio between the different trader types, such that R = MAA:SHVR:ZIC:ZIP.

If we set $NEqR$ to 3, that means when the ratio R is equal, it will be 3:3:3:3 on the buyer side (and 3:3:3:3 on the seller side), so there will be $S{\cdot}NEqR$=3x4=12 traders on each side, so P=24. When we say that we are exhaustively varying the ratios, this means that for any given number of buyer or seller traders $P/2$, we test all possible ratios for that given $P/2$, sweeping the counts of each trader-type in R through all valid nonnegative integers. Taking again the P=24 example from our experiments in Section 5, this means starting with R=0:0:0:12 on each side of the market, running T trials (independent market sessions) at that ratio, then running T trials at R=0:0:1:11, a further T at R=0:0:2:10, and so on systematically adjusting all counts in the ratio, through the equal-ration case of R=3:3:3:3 and on to the final ratio of R=12:0:0:0. The combinatorics are quite explosive: for any particular values of S and $NEqR$, total number of different viable ratios R is given by:

$$R = (S \cdot NEqR + S - 1)! \,/\, ((S \cdot NEqR)! \cdot (S\text{-}1)!)$$

Hence the total number of market sessions that need to be run for any one value of $NEqR$ is $R \cdot T$. This gets quite big, quite fast: e.g., with S=4, $NEqR$=4, and T=100, we have $R \cdot T$=96,900. And to rigorously explore population-size effects we sweep $NEqR$ through a range of values.

4.2 Varying Supply/Demand Schedules

The review in Section 2.3 demonstrated that typically the supply and demand schedules induced by the experimenter (via the choice of limit prices in the traders' assignments) are such that the equilibrium price P_0 is either constant for the duration of the experiment, or undergoes one or more step-changes, (price shocks) in the course of the experiment, jumping from one constant value to another. Much of the work reviewed in Section 2.3 also involves periodic replenishment of all traders' assignments, dividing the experiment into a number of trading "periods". While this style of experiment design will certainly have been most convenient for Vernon Smith when he was running his early experiments, entirely manually, in the late 1950s and 1960s, once everything is under computer control it seems a curious thing way to organise things, especially given the observation that almost every real-world market of interest is quite clearly not fixed at a constant equilibrium price, undergoes step-changes in P_0 only very rarely (if at all) and that in the course of a trading day for any reasonably liquid tradeable asset the flow of orders (i.e., trader assignments) into the market is not neatly periodic but instead is best modelled as a stochastic process, with random interarrival times.

Fortunately, BSE offers the experimenter a lot of control over the supply and demand schedules (SDSs) used in any one experiment. Traditional ExpEcon constant-P_0 SDSs can easily be specified in BSE, with or without step changes so that P_0 jumps from one constant value to another, but BSE also allows for constantly-varying SDSs to be specified, driven by closed-form functions or by look-up tables (LUTs) which each specify an offset value, denoted $P_0^+(t)$ that is added to P_0 at time t during the experiment. To study the response of MAA to continuously-varying P_0, we used LUTs of real-world financial-asset intra-day time-series drawn from a range of asset classes, as described in Section 5.2: in these experiments the supply and demand curves were totally flat, with all supply-curve limit prices $P_S=P_0^+(t)$ + \$0.95 and all demand-curve prices $P_S=P_0^+(t)$ + \$1.05. In contrast, in the S'62 experiments reported in Section 5.1, the SDSs were constant ($P_0^+(t)$=0 for all time) and symmetric over the range \$0.10 to \$1.90.

5 RESULTS

5.1 Experiments in the Style of Smith'62

Our first set of S'62 experiments explores the profitability of MAA, SHVR, ZIC, and ZIP in market experiments run within BSE but modelled as closely as possible on Smith's original experiments: individual traders are either buyers or sellers; trading happens in discrete periods ("days"), with all traders' assignments of buy and sell orders being simultaneously replenished at the start of each period. The SDS are such that the underlying equilibrium price P_0 is held fixed for the duration of the experiment, or is subjected to one or more step-change "price shocks" which always occur at the start of a trading period. Figure 1 shows a comparison of the results, expressed as *average profit per trader* (APPT) from markets populated by a mix of MAA and ZIP traders, in ratios varying from roughly 5%:95% through 50%:50% to 95%:5%.

This is exactly the kind of comparison that is usually reported in the trading-agent literature. From Figure 1 it is clear that, for both MAA and ZIP, when either strategy is in the minority (<50% of the traders in the population), the profit scores are roughly the same; but as the proportion increases beyond 50%, MAA's profit scores are significantly better than those of ZIP. The data in Figure 1 should cause no surprises to anyone familiar with the literature surveyed in Section 2.3: this is confirmation that MAA can outperform ZIP, which is to be expected from the results and analysis previously published by Vytelingum and by De Luca and Cliff. AA's

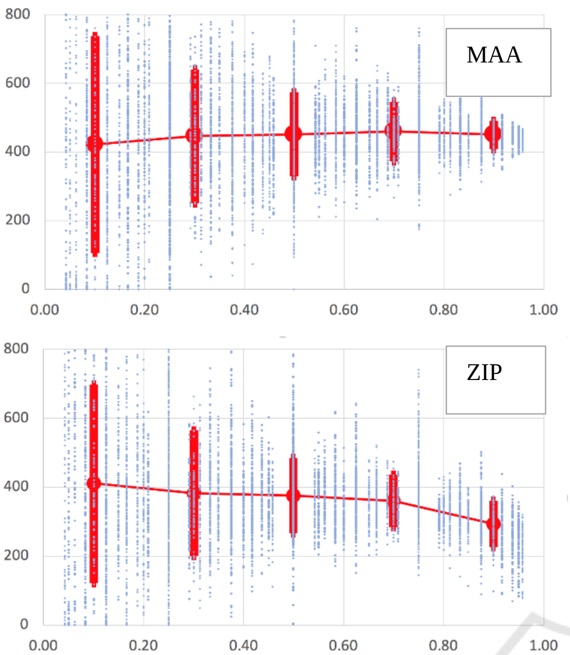

Figure 1: Results from 7,800 separate S'62 market sessions pitting some number of MAA traders against some number of ZIP traders, in markets where the total number of traders varies from 8 (4 buyers + 4 sellers) to 48 (24+24). Upper graph is MAA results; lower graph is ZIP. Horizontal axis is percentage of that strategy within the population; vertical axis is average profit per trader (APPT). Small blue markers are results from individual market sessions; large solid-red markers show the mean, with error bars to plus and minus one standard deviation, for quintile bins (i.e., 0-20%, 20%-40%, etc). For both strategies, when in the minority the results are broadly similar, but when MAA is in the majority it scores significantly higher profit than MAA.

dominance is clearest when it is trading in markets where most other traders are *also* using the MAA strategy. Thus far though, the king retains the crown.

While Figure 1 shows the effect of varying the proportion of two trading strategies in a two-trader market, summarising results from 7,800 separate market sessions, the number of different situations studied there is very small in comparison to the space of all viable ratios across some reasonable range of population sizes. Figure 2 illustrates aggregate statistics from 546,000 market sessions that exhaustively explore that whole space. Here the ratios of four strategies are systematically varied over all viable values (so this includes the data shown in Figure 1, where the MAA:SHVR:ZIC:ZIP ratio R was restricted to match n:0:0:m). Figure 2 shows APPT for the four trading strategies plotted as *NEqR* varies over the range [1,6] (i.e., total number of traders in the range [8, 48]), with T=100 at each ratio.

It is clear from Figure 2 that as *NEqR* increases there is a slight reduction in variance; and although the mean values of the four trader types differ, these differences are tiny in comparison to the standard deviations: when measured by APPT there are no major differences when the whole space is sampled.

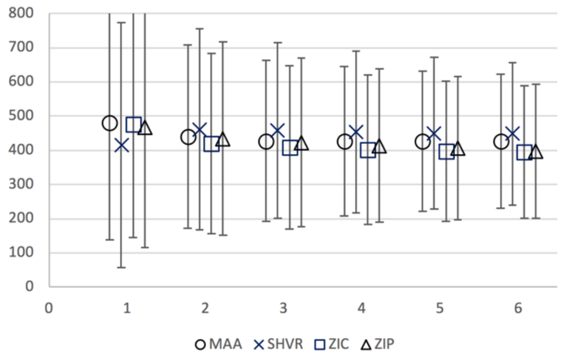

Figure 2: Results from 546,000 separate S'62 market sessions with periodic replenishment of traders' assignments. Horizontal axis is *NEqR* values; vertical axis is APPT, with error bars at plus and minus one standard deviation. The explosive combinatorics of the exhaustive sweep through all combinations of ratios of the four trader types for any specific value of *NEqR* means that the number n of discrete experiments summarised by each marker on the graph for *NEqR*=1,2,…,6 are respectively: n=2,000; 12,000; 36,400; 81,600; 154,000; and 260,000.

This may seem like a counterintuitive result: in these experiments the zero-intelligence ZIC and SHVR are scoring just as well as MAA and ZIP. It can be explained by reference to three factors: choice of metric; heterogenous trader populations; and experiment design. On the choice of metric: if we had reported the traditional metrics of Smith's α or PD, the differences between strategies would have been more clear; transaction prices in markets populated by ZIC and SHVR do show increased α (i.e., RMS deviation of transaction prices from the theoretical P_0 value) and PD (i.e., differences between actual profit accrued, and profit expected if all transactions took place at the P_0 price), but as was argued above, the bottom line in a real-world trading environment is actual profit. On the heterogeneity of the trading population: in almost all of the market sessions summarised in Figure 1, the "dumb" traders playing the SHVR strategy can, in essence, get a free-ride from the AI/ML in MAA and ZIP: as traders playing those "intelligent" strategies post prices, SHVR traders can parasitically jump one cent better, immediately posting a better price, positioning themselves at the top of the LOB. On the experiment design, this classic SDS where the P_0 is static for the entire experiment does not exactly provide the most taxing environment in which to trade;

Table 1: Results table for S'62 experiments with periodic (upper sub-table: PR) and continuous (lower sub-table: CR) replenishment of trader assignments for *NEqR*=1…5. Each sub-table shows the sample mean μ and standard deviation σ of the APPT scores for the four strategies. The μ values of the four strategies are so tightly clustered, relative to the σ values, that the differences between the μ values are of no consequence. The column *N* is the total number of separate market sessions run for that sub-table.

		MAA	SHVR	ZIC	ZIP	N
S'62 PR	μ	428	452	402	412	286,000
	σ	217	234	217	223	
S'62 CR	μ	431	462	409	414	286,000
	σ	223	231	245	218	

in the rest of this paper we test the strategies in more challenging environments, and the differences in their performance come much more starkly into view. Our second set of S'62 experiments uses the same SDS as our first, but we switch from periodically updating all trader's assignments at the same time, at the start of each trading period or "day", and instead have assignments made continuously, arriving at random during the course of the experiment which has the same overall duration but is no longer sensibly spoken about as being divided into distinct periods. To save space, we will move from graphical presentation of results to tabular. Table 1 presents the numeric values shown graphically in Figure 2 from our periodic-replenishment (PR) experiment, along with the corresponding values from the same experiment, same SDS, run with continuous-replenishment (CR). As can be seen, the move to CR (which is much closer to real-world markets) has no impact on the rank-ordering of the strategies.

5.2 Real-World-Dynamics Experiments

To explore whether AA dominates in more realistic environments, a set of experiments were run where the market's underlying equilibrium price was varied dynamically using an appropriate $P_0^+(t)$ function with the SDS, as described in Section 4.2.

A first set of experiments, involving 858,000 simulated market sessions, was run where $P_0^+(t)$ was generated from a closed-form sinusoidal function. Results from these experiments (not presented here, due to space constraints) indicated that MAA did not do well in such circumstances, but were open to the criticism that the $P_0^+(t)$ functions involved were too artificial, too unlike real-world dynamics. For that reason, a second set of 1,716,000 experiments were run, referred to here as *Real-World Dynamics* (RWD). In the RWD experiments, $P_0^+(t)$ was determined by a LUT of intra-day price movements of a specific real financial asset on a particular date. In an attempt at mitigating any biases in the dynamics of a particular asset class, we ran sets of RWD experiments using intra-day price data from six different classes of asset: an equity; a foreign-exchange (FX) currency-pair; a government bond; a metal; a commodity; and an aggregate index. In any one RWD experiment the intra-day price time-series at one-minute resolution for a specific asset on a specific date was read into BSE and then normalised on the time and price axes to give a LUT that could return a P_0^+ value at any point in the duration of the experiment, with prices in the range [0, 80] for ease of comparison across the six different asset classes.

In the results shown here, the RWD-Equity experiment uses prices of IBM stock on 08/31/17; RWD-FX uses price data for GBP-USD (i.e., "Cable") on 09/11/17; RWD-Bond uses prices of the US Government 10yr Treasury Note on 09/12/17; RWD-Metal uses data for Copper on 09/12/17; RWD-Commodity uses spot Brent Crude Oil on 15/29/18; and RWD-Index uses NASDAQ on 09/13/17. In each case, 1-minute intraday price data was taken from the free samples available at the website of *BacktestMarket.com;* the date chosen for use in each asset class is simply the first date available in the BacktestMarket sample data, and hence is arbitrary. For each asset-class of RWD experiment we ran an exhaustive sweep where *NEqR* values were varied over the range [1,5] (i.e., markets with *P*=8, 16, 24, 32, and 40 traders, always 50% buyers and 50% sellers), where for each *NEqR* value all possible ratios R of trading strategies were tested, and where for any specific *(NEqR,* R*)* combination we executed *T=100* independent simulated market sessions. This required a total of 1,716,000 market simulations across the six asset classes.

Figure 3 shows summary data from these experiments: as is clear, MAA is again the 3rd-ranked strategy, and again it performs significantly worse than either SHVR or ZIP. After viewing these results, there is no reasonable way that MAA can continue to be seriously considered as the best-performing published strategy.

6 DISCUSSION AND CONCLUSIONS

The results in Figure 1 and Table 1 confirm what anyone familiar with the CDA trading-agent literature would reasonably claim to already know: AA, when appropriately modified to work in a LOB-based

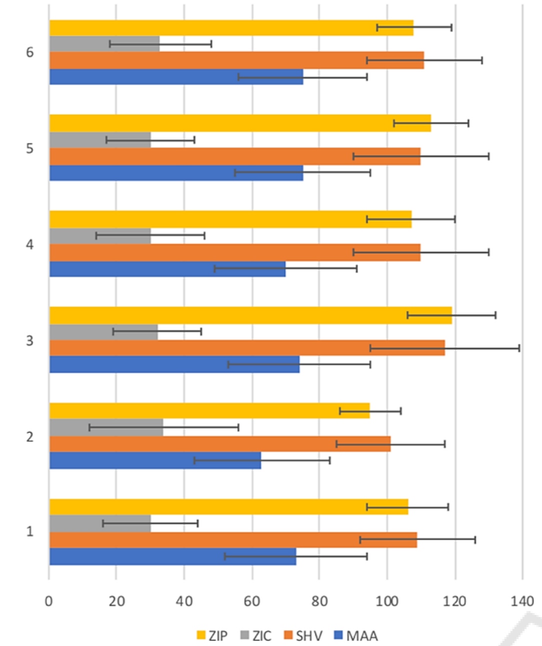

Figure 3: Summary of results from RWD experiments across six asset classes: this chart summarises results from 1,716,000 separate market sessions. Horizontal axis is average profit per trader (APPT). Results are grouped by asset-class, with group-numbers on the vertical axis. Group 1 (at bottom) are from bond-price experiments; Group 2 are from commodity-price; Group 3 from equity-price; Group 4 from metal-price; Group 5 from FX prices; and Group 6 from index prices. Bars show mean APPT with error-bars indicating plus and minus one standard deviation.

CDA, and when tested in the kind of simple market environment as has traditionally been used in the literature, scores just as well as well-known other trading strategies and is not dominated by them.

But the results in Figure 3 blow a major hole in the status quo: merely by altering the nature of the market environment to have continuous stochastic replenishment (which is surely what happens in real markets) and to have the equilibrium price P_0 continuously varying over time (which is also surely what happens in real markets), the results we get from MAA are very poor indeed. On the basis of these results, it is manifestly no longer correct to claim that AA/MAA is the best-performing trading strategy known in the published literature. How well the previous title-holder, i.e. GDX, fares in RWD experiments is an obvious line of further enquiry.

It seems very hard to avoid the conclusion that AA's success as reported in previous papers is largely due to the extent to which its internal mechanisms are designed to fit exactly the kind of experiment settings first introduced by Vernon Smith: AA is very well suited to situations in which all assignments are issued to all traders simultaneously, and in which the equilibrium price remains constant for sustained periods of time, with only occasional step-change "shocks". Real markets are not like this, and when AA is deployed in the more realistic market setting provided by BSE, its dominance disappears.

But surely the broader lesson here is that we should not allow ourselves to be seduced by results from small-scale studies in minimally simple approximations to real-world markets. Smith developed his experimental methods in the late 1950's when there were no realistic alternative ways of doing things. Running experiments with human subjects is laborious and slow, but experiments in electronic markets populated entirely by robot traders can proceed in appropriate simulators at speeds much faster than real-time, and are "embarrassingly parallelizable": the experiments reported in this paper took a couple of weeks; if I'd used more virtual machines they could have been done in a couple of days or even in a couple of hours.

At this point in time, 20% of our way into the 21st Century, surely trading-agent researchers should collectively abandon the simple minimal test-beds that worked well for Vernon Smith in the middle of the 20th Century and instead start to tolerate the minor inconvenience of running very large numbers of trials on reasonably accurate simulations of realistic market situations: the methods used here should be the norm, not the exception. The availability of open-source public-domain exchange simulators such as BSE, OpEx, and ExPo, coupled with readily available cheap cloud-computing for doing the necessary processing, means that there are now really no excuses for not doing so.

REFERENCES

Bse, 2012. "Bristol Stock Exchange" GitHub repository at https://github.com/davecliff/BristolStockExchange

Cliff, D. 1997. "Minimal-Intelligence Agents for Bargaining Behaviours in Market-Based Environments". HP Labs Technical Report HPL-97-91.

Cliff, D. 2018. "An Open-Source Limit-Order-Book Exchange for Teaching and Research." *Proc. IEEE Symposium on Computational Intelligence in Financial Engineering (CIFEr).* SS-1296, pp.1853--1860.

Das, R., Hanson, J., Kephart, J., and Tesauro, G. 2001. "Agent-Human Interactions in the Continuous Double Auction". *Proceedings IJCAI-2001*, pp.1169-1176.

De Luca, M. and Cliff, D. 2011a. "Agent-Human Interactions in the CDA, Redux". *Proc. ICAART2011*.

De Luca, M. and Cliff, D. 2011b. "Human-Agent Auction Interactions: Adaptive-Aggressive Agents Dominate."

Proc IJCAI-2011, pp.178-185.
De Luca, M., 2015. *Adaptive Algorithmic Trading Systems.* PhD Thesis, University of Bristol, UK.
Gjerstad, S. and J. Dickhaut, J. 1998, “Price Formation in Double Auctions”, *Games and Economic Behavior*. 22(1):1-29.
Gode, D. and Sunder, S. 1993. “Allocative efficiency of markets with zero-intelligence traders”, *Journal of Political Economy*, 101(1):119-137.
le Calvez, A. and Cliff, D. 2018. “Deep Learning can Replicate Adaptive Traders in a LOB Financial Market.” *Proc. IEEE Symp. on Comp. Intelligence in Financial Eng. (CIFEr).* SS1070, pp.1876--1883.
Rodgers, K. 2016. *Why Aren’t They Shouting? A Bankers Tale of Change, Computers, and Perpetual Crisis.* RH Business Books / Cornerstone Digital.
Smith, V. 1962. “An experimental study of competitive market behavior”, *JPE* 70(2):111-137.
Stotter, S., Cartlidge, J., and Cliff, D. 2013. “Exploring assignment-adaptive (ASAD) trading agents in financial market experiments”, *Proc. ICAART*, 1:77-88.
Stotter, S., Cartlidge, J., and Cliff, D. 2014. “Behavioural investigations of financial trading agents using Exchange Portal (ExPo).” In N. Nguyen *et al.* (eds), *Trans. Comput. Collective Intell. XVII*. pp. 22-45.
Tesauro, G. and Das, R. 2001. “High-performance Bidding Agents for the Continuous Double Auction”. *Proc. 3rd ACM Conf. on Electronic Commerce*, pp.206-209.
Tesauro, G. and Bredin, J., 2002. “Sequential Strategic Bidding in Auctions using Dynamic Programming”. In *Proceedings AAMAS 2002*.
Vach, D. 2015. *Comparison of Double Auction Bidding Strategies for Automated Trading Agents.* MSc Thesis, Charles University in Prague. https://bit.ly/2rk1clV
Vytelingum, P. 2006. *The Structure and Behaviour of the Continuous Double Auction*. PhD Thesis, University of Southampton, UK. https://bit.ly/2Sw78nv
Vytelingum, P., Cliff, D., and Jennings, N. 2008. “Strategic Bidding in CDAs”. *Art. Intell.*, 172(14): 1700-1729.