The Risk-averse Profitable Tour Problem

Maria Elena Bruni[1], Lorenzo Brusco[2], Giuseppe Ielpa[2] and Patrizia Beraldi[1]
[1]*Department of Mechanical, Energy and Management Engineering, Unical, Italy*
[2]*Department of Mathematics and Computer Science, Unical, Italy*

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Abstract: In this paper, we tackle the risk-averse profitable tour problem with stochastic costs and risk measure objectives. This problem aims at determining a tour that maximizes the collected profit minus the total travel cost under a risk-averse perspective. We explore efficient implementations of a genetic algorithm and a tabu search method to solve the problem when the conditional value at risk and entropic risk measures are used. The computational study shows the superiority of the genetic algorithm over the tabu search on a set of instances adapted from the TSP library.

1 INTRODUCTION

Traveling Salesman Problems with Profits (TSPP) are single-vehicle routing problems with two conflicting objectives, the maximization of the total collected profit and the minimization of the total route cost. Depending on the way these objectives are combined, three classes of TSP with profits can be distinguished (Feillet et al., 2009). When both criteria are combined linearly in the objective function, the problem is the so-called Profitable Tour Problem (PTP) introduced by (Dell'Amico et al., 1995). When the profit is maximized and a constraint is added to the problem, limiting the total route cost from above, the problem is either referred to as the Orienteering Problem (OP) or the Selective Traveling Salesman Problem (STSP). On the other hand, when the objective is to minimize the costs and a constraint on the collected profit is added to the model, the problem is called the Prize Collecting TSP (PCTSP). In all these three different variants, in contrast to the original TSP, the visit of all customers is no longer mandatory and a specific profit is collected when a customer is visited. The problem belongs to the class of vehicle routing problems with profits, a flourishing literature stream that has attracted the attention of the operations research community in the last ten years (Beraldi et al., 2015a; Bruni et al., 2018b; Beraldi et al., 2019). Routing problems with profits arise in a number of application areas. In particular, there are many applications for which the PTP would be an appropriate model either as in it-self or as a key subproblem of more involved problems. For instance, considering some agencies whose service technicians must visit geographically dispersed customers, it is easy to recognize that each technician can be scheduled to service a subset of customers. In choosing this subset of customers, one may consider priorities for specific customer visits (very often depending on the profit achievable) as well as the estimated costs of the service (including the trip to the customer). The tourist trip design problem is another example, referring to a route-planning problem for tourists interested in visiting multiple points of interest. The main objective of the problem is to select points of interest that maximize tourist satisfaction, while taking into account a multitude of parameters among which the tourist traveled distance. In a post-disaster setting, such as one following an earthquake or a flood, the goal of a search and rescue team is to identify damaged or collapsed structures in the affected area to rescue as many survivors as possible, trading-off the priority with the total service time (Bruni et al., 2018a).

Assuming deterministic data is often unreasonable in real applications. Travel times, travel cost and even profits are seldom known in advance and very often can be at best be estimated. A common approach under uncertainty is to consider a risk neutral viewpoint, notably implemented through the minimization of the expected value of the random objective function (Bruni et al., 2014; Beraldi et al., 2015b). However, often, the decision maker is risk-averse and, hence, more interested in hedging against extreme realizations. In this paper, we consider the conditional value at risk (CVaR) and the entropic value at risk (EVaR) measure. While the CvaR plays a central role amongst coherent risk measures and is the most used risk measure in practical applications (Beraldi et al.,

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2012), the entropic risk measure has been recently considered an appropriate measure in routing problems (Cominetti and Torrico, 2016). In this paper, we provide a risk-averse formulation for the PTP assuming that the route cost is uncertain and propose a genetic algorithm and a tabu search to solve it. To th best of our knowledge this is the first contribution dealing with the PTP under a risk averse perspective. The rest of the paper is organized as follows. The next section discusses the related work. Section 3 recalls the PTP and introduces the problem under risk. Section 4 defines the algorithmic approaches proposed to solve the problem. Section 5 presents the computational results, and finally, Section 6 concludes the paper.

2 LITERATURE REVIEW

Among the routing problems with profits the problem that has been studied in depth is the OP. In (Millar and Kiragu, 1997) a time-based formulation and an upper bound on the number of vertices visited in an optimal solution were proposed. A branch-and-bound algorithm using a Lagrangean relaxation was proposed in (Ramesh et al., 1992), whereas a branch-and-cut algorithm using several families of valid inequalities was presented in (Fischetti et al., 1998). Several heuristics were proposed for the OP. In (Chao et al., 1996) a heuristic is proposed and compared with the previously published ones, all based on local ascent schemes. A metaheuristic approach based on Tabu Search was proposed in (Gendreau et al., 1998). The instances tested had up to 300 vertices.

A number of studies have addressed the deterministic PCTSP, proposing exact and heuristic algorithms. A polyhedral study of the capacitated version of the PCTSP, can be found in (Balas, 1995). Bounding procedures, based on different relaxations, were proposed for the same problem with penalties in (Fischetti and Toth, 1988), whereas an exact branch-and-cut algorithm able to solve instances with more than 500 vertices was proposed in (JF. Berube and Potvin, 2009). A branch-and-cut algorithm based on a directed graph model where several state-of-the-art methods are combined was proposed for the the Steiner tree problem in (Leitner et al., 2017; Klau et al., 2004). A Lagrangian heuristic, that starts from a lower bound to the problem and makes the solution feasible was proposed in (Dell'Amico et al., 1998).

Even though there is considerable previous work on models and methods to incorporate uncertainty in combinatorial optimization problems and in vehicle routing problems, a few contributions exist on profit-based routing problems at the presence of uncertainty. The stochastic STSP was introduced in (Tang and Miller-Hooks, 2005), where the aim is to find a tour with a maximum objective value consisting of total reward minus total travel cost. A chance constraint imposes that the total duration of the tour should be lower than a threshold. (Campbell et al., 2011) introduced a stochastic variant of the OP in which travel and service times are stochastic and a penalty is incurred for the nodes not serviced at the end of the day. The objective is to maximize the total expected profit minus the penalty for the unmet demands. In (İlhan et al., 2008) an OP where the collected prizes are subject to uncertainty is considered. The objective is to maximize the probability of collecting more than a specified target prize level. For this problem the authors propose a parametric exact algorithm and a genetic algorithm. Another stochastic variant with recourse of the OP with hard capacity constraints was introduced by (Evers et al., 2014). The authors proposed a sample average approximation and a tailored heuristic. To the best of our knowledge, neither specific exact approaches, nor computational analysis of heuristic algorithms have been specifically proposed for the PTP, probably due to its simple structure (Archetti et al., 2013). Hence, the present paper contributes to the literature proposing a risk-averse variant of the PTP as well as tailored solution approaches to solve the resulting complex model.

3 PROBLEM FORMULATION

In this section, we first define the problem and introduce the notation used throughout the paper. Afterwards, we present a mathematical formulation for the problem. A complete graph $G := (V,E)$ is given, where $V = \{0,...,n\}$ represents the set of vertices in $V_0 = V \backslash \{0\}$ correspond the set of customers, and E is the edge set. Whenever customer i is visited, a profit p_i is collected. The profit of each customer can be collected at most once. We assume that the time to serve a customer is negligible and that to each edge $(i, j) \in E$ is associated a cost c_{ij}. The objective is to find a server's route which maximizes the total not profit collected over the graph, defined as the total profit minus the total route cost. Let us denote by y_i, $i \in V \setminus 0$, the binary variable indicating whether the corresponding client i is served or not, and by x_{ij}, $(i, j) \in E$, the binary variable taking the value 1 if the corresponding arc is traversed and 0 otherwise. The deterministic PTP can be formulated as follows.

$$max \sum_{i \in V_0} p_i y_i - \sum_{(i,j) \in E} c_{ij} x_{ij} \tag{1}$$

$$\sum_{j \in V} x_{ij} = y_i \quad i \in V \tag{2}$$

$$\sum_{i \in V} x_{ji} = y_i \quad i \in V \tag{3}$$

$$y_0 = 1 \tag{4}$$

$$u_i + 1 - |V|(1 - x_{ij}) \leq u_j \quad i, j \in V, i \neq j \tag{5}$$

$$x_{ij} \in \{0,1\} \quad (i,j) \in E \tag{6}$$

$$y_i \in \{0,1\} \quad i \in V \tag{7}$$

$$u_i \geq 0 \quad i \in V. \tag{8}$$

Equalities (2) are degree constraints imposing that for each vertex the in-degree and out-degree has to be the same and equal to 1 in case of visit. Constraints (5) are the Miller-Tucker-Zemlin constraints preventing subtours. These constraints require the introduction of additional continuous variables $u_i, i \in V$, where variable u_i represents the arrival time at node i.

Let consider now uncertain costs $\tilde{c}_{ij}$ on every edge such that the total tour cost $X = \sum_{(i,j) \in E} c_{ij} x_{ij}$ is a random variable with cumulative distribution function F_X, defined on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F}$ is a $\sigma - algebra$ of subsets of Ω. Instead of considering a risk neutral approach, we consider in this work the solution of the PTP that minimizes a risk measure associated with the total profit, i.e. aims at maximizing a given safety measure. Formally, a risk measure is a map $\rho : \mathcal{X} -> \mathcal{R}$ that attaches a scalar value to each random variable $X : \Omega -> \mathcal{R}$, whose moment-generating function $M_X(z)$ exists for all $z \geq 0$. In our study, we consider two specific risk measures for the random cost X: the CVaR and the EVaR.

CVaR

Before presenting the CVar, let us define the value-at-risk (VaR) as the first quantile function of the distribution function F. The VaR with confidence level $1-\alpha, \alpha \in (0,1)$ can be found by solving the following problem

$$VaR_{1-\alpha} = \inf_{\eta}(\eta | F(\eta) \geq 1 - \alpha)$$

and is equivalent to the left-continuous inverse of the cumulative distribution function. The VaR is the smallest value of X if we exclude worse outcomes whose probability is less than α. More formally, the VaR is defined such that the probability that the random variable X is greater than VaR is less than or equal to α while $1 - \alpha\%$ of the cost realizations are equal or below the VaR (since $F(\eta) = \mathbb{P}(X \leq \eta)$). Hence, when $alpha$ is small, we are confident at the $1 - \alpha$ probability that the cost will not exceed $\eta*$, the VaR.

The CVaR is an important coherent risk measure that was introduced and studied recently in (Rockafellar and Uryasev, 2002). The CVaR with confidence level $1 - \alpha$ is defined as follows:

$$CVaR_{1-\alpha} = \mathbb{E}[X | X \geq VaR_{1-\alpha}].$$

If F_X is continuous, then we have

$$CVaR_{1-\alpha} = \frac{1}{1-\alpha} \int_{\alpha}^{1} VaR_{1-t} dt \tag{9}$$

The CvaR is consistent with the second-degree stochastic dominance and it is coherent in the terminology of (Ogryczak and Ruszczyński,). Moreover, depending on the choice of α, $\mathbb{E}_{\gamma}(E(\omega))$ can be used to represent a broad spectrum of preferences ranging from the most conservative risk adverse position ($\alpha = 0$) to risk neutrality ($\alpha = 1$).

Entropic VaR. The EVaR is a coherent risk measure introduced by (Ahmadi-Javid, 2012a; Ahmadi-Javid, 2012b). The entropic EVaR of X with confidence level $1 - \alpha$ is defined as follows:

$$EVaR_{1-\alpha} := inf_{z>0}\{a_X(\alpha, z)\} =$$

$$= inf_{z>0} z^{-1} ln(M_X(z)/\alpha).$$

It can be shown that EVaR is e the tightest possible upper bound for the value at risk VaR and the CVaR, at the same level of confidence which means that EVaR is known to be more risk averse compared to others. These results are obtained from the Chernoff inequality. In particular, the Chernoff inequality for any constant a is

$$\mathbb{P}(X \geq a) \leq e^{-za} M_X(z).$$

By solving the equation $e^{-za} M_X(z) = \alpha, \alpha \in (0,1]$ we obtain $a_X(\alpha, z) := z^{-1} ln(M_X(z)/\alpha)$. In the case of continuous distributions, the evaluation of the EvaR, as well as CvaR, requires the computation of the convolution of random variables and, depending on the chosen probability distribution, this task can be a straightforward procedure or a complex operation.

For a normally distributed random variable $X \sim \mathcal{N}(M, \Sigma)$ we can derive a deterministic equivalent for both CVaR and EVaR., In particular, it can be shown that

$$CVaR_{1-\alpha} := M + f(z(\alpha))\Sigma,$$

where $f(\cdot)$ is the probability density function of $\mathcal{N}(0,1)$ and $z(\alpha)$ is the α-quantile of $f(\cdot)$ and

$$EVaR_{1-\alpha} := M + \sqrt{-2ln(\alpha)}\Sigma.$$

Assuming that the random costs are normally distributed with mean $\bar{c}_{ij}$ and variance σ_{ij}^2, and considering that the normal distribution is closed under affine transformations, the total profit X is again a random variable with expected value $\sum_{i \in V_0} p_i y_i - \sum_{(i,j) \in E} \bar{c}_{ij} x_{ij}$ and variance $\sum_{(i,j) \in E} \sigma_{ij}^2 x_{ij}^2$. Hence, the risk averse PTP with the CVaR risk function can be written as

$$\sum_{i \in V_0} p_i y_i - \sum_{(i,j) \in E} \bar{c}_{ij} x_{ij} + f(z(\alpha)) \sqrt{\sum_{(i,j) \in E} \sigma_{ij}^2 x_{ij}^2)}$$

$$s.t. (2) - -(8)$$

and the PTP with the EVaR risk function as

$$\sum_{i \in V_0} p_i y_i - \sum_{(i,j) \in E} \bar{c}_{ij} x_{ij} + \sqrt{-2ln(\alpha)} \sqrt{\sum_{(i,j) \in E} \sigma_{ij}^2 x_{ij}^2)}$$

$$s.t. (2) - -(8)$$

To assess the importance of incorporating cost fluctuations in the problem, we present in Figure 1 a fictitious small example, including five potential nodes to be visited. The ordered pair of over each edge represents the expected cost and its standard deviation, respectively. The value under each node represents the revenue collected while visiting the node.

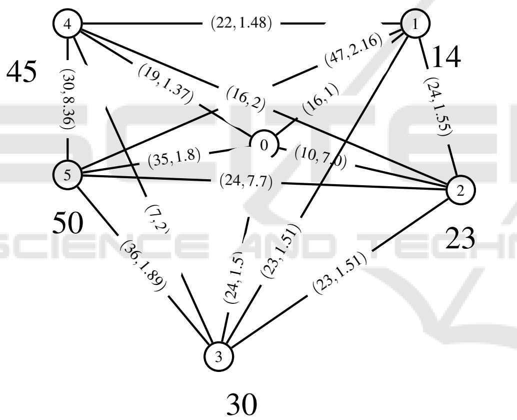

Figure 1: Network of example.

The solution of the deterministic PTP is the the tour (0-2-5-4-3-0) with optimal objective function value of 53 and a total variance of 186.4. When we consider the EVaR risk measure with $\alpha = 0.01$, we obtain an optimal path (0-4-3-5-2-0) with a total variance of 119.5, which is much lower that the standard deviation of the risk-neutral solution. In this case, the decision-maker is willing to trade-off some profit against less risky solutions.

The PTP is $NP-$hard even in the deterministic case. The injection of uncertainty increases the complexity of the problem, preventing the exact solution within a reasonable time limit. In what follows, we present two heuristic approaches for the risk averse PTP. One is based on genetic algorithm and the other is a tabu search heuristic.

4 HEURISTIC SOLUTION APPROACHES

The Genetic Algorithm

A Genetic Algorithm (GA) is an evolutionary algorithm that mimics the natural selection. A standard GA starts with a population of encoded solutions (chromosomes), which are initially, randomly generated. In our case, tours are encoded into two chromosomes as an ordered list of vertices serviced, as usual in the TSP, and a list of not visited nodes. Each individual is then evaluated on the basis of the fitness function ϕ which is simply the evaluation the objective function associated to the solution corresponding to the encoding of the individual. We generate the initial population randomly generating a number of nodes to be visited (number of genes in the first chromosome) and then by picking the value of each gene from the standard uniform distribution in the range [0,n]. The second chromosome is simply the list of nodes not included in the first one. After the construction of the initial population, the GA generates a new set of individuals from the parent population, applying the crossover operator on the first chromosome, which recombines parts of the parent information. In our case the order crossover operator has been applied. A tournament selection method for a parent selection is used, which begins by forming two teams of chromosomes. Each team consists of ten individuals randomly drawn from the current population. The best individuals, selected from each of the two teams, are then chosen for crossover operations. As such, two offsprings are generated and entered into the new population. The best solution at each iteration is improved by applying the two-opt heuristic.

As a diversification mechanism, the GA may employ a mutation operator which modifies a child chromosome with a given probability. We implemented two possible kinds of mutation: inserting a new gene or removing an existing gene. In particular, for each individual, the following procedure is executed. For each node $i \in V$ a random number is generated between 0 and 1. If the random number is below a given mutation probability π_{mut} and the node is present in the individual, it will be removed from the tour, otherwise it will be added in the best position in the tour. After the generation of the offspring, some chromosomes in both the parent population and the offspring are eliminated according to their relative fitness function values. The remaining chromosomes form a new population. The scheme of the GA is reported in Algorithm 1.

Algorithm 1: The GA pseudo-code.

Input: π_{mut}
Initialization: $best, currentbest := null$; $P^{curr}, P^{best} = \emptyset$.
Create an initial population of $|V|$ individuals and store them in P^{curr}. Evaluate the fitness of the population.
repeat
  $P^{it} = P^{curr}$
  $P^{cross}, P^{mut} = \emptyset$
  Apply the tournament selection method to the population. Let the parents be the best inidivduals in each tournament.
  Apply the order crossover operator to the parents, obtaining two new individuals and apply the generational replacement.
  for $i \in P^{it}$ **do**
    for $g = 1, \ldots, |V|$ **do**
      Generate a random value r in (0,1).
      if $r <= \pi_{mut}$ **then**
        if *g is in the chromosome list of the individual i* **then**
          remove it
        end
        add g in the best position
      end
    end
  end
  it:=it+1
  $currentbest = argmin_{i \in P^{curr}} \phi_i$
  if $\phi_{currbest} > \phi_{best}$ **then**
    Apply two-opt. Update ϕ_{best}
  end
until *a given termination criterion is met*;
Return ϕ_{best}

The TABU Search Algorithm. The first heuristic is a tabu search heuristic (TS), inspired by the one proposed in (Gendreau et al., 1998). This kind of metaheuristic has been proved to be successful in solving difficult routing problems (Guerriero et al., 2013). Generally speaking, the TS systematically explores the solution space by moving at each iteration from the current solution s to the best one amongst its neighbours. To avoid cycling, solutions possessing some attributes of recently explored solutions are temporarily declared tabu (i.e. forbidden) and the number of iterations an attribute remains tabu is denoted with θ.

The initial tour is built by a construction heuristic which forms a tour of length $\frac{n}{2}$. First, the two highest profit vertices are included in the tour T and afterwards, in each successive iteration and until the desired length is reached, two adjacent vertices are randomly determined in the tour (let say i and k) and a city $j \notin T$ having the minimal ratio $(\bar{c}_{ij} + \bar{c}_{jk} - \bar{c}ik)/p_j$, is added. The whole procedure is repeated for five times, and best tour is retained. Before starting with the tabu iterations, several partitions of the node set V_0 are defined, each containing one or more clusters of vertices. The first partition contains n clusters, each containing one node. In any successive partition two clusters (R, S) yielding the minimum proximity measure Δ, are merged (in the first iteration the clusters are singletons). The proximity measure between two clusters R and S is defined as $\Delta(R,S) = \frac{2}{|R||S|} \sum_{i \in R, j \in S} \bar{c}_{ij} - \Gamma(R) - \Gamma(S)$, where $\Gamma(R) = \frac{1}{|R|(|R|-1)} \sum_{i,j \in R} \bar{c}_{ij}$ if the cluster has at least two nodes and zero otherwise. Only partitions with a number of nodes equal to $n+1-1, n+1-[n/2], n+1-[2n/3], n+1-[3n/4], \ldots, n+1-[9n/10]$ are retained. Then one partition is selected randomly and each cluster belonging to the partition is evaluated. The best cluster will be then selected to form the neighbours of the current solution, obtained by two possible moves. Either a move consists in inserting a cluster of nodes in their best position in current solution or in removing a chain of nodes (a set of adjacent nodes) from the current solution.

Each cluster is evaluated for insertion on the basis of the ratio of added profit over added distance. In particular, the gravity centre is first computed for all the clusters and a preliminary move evaluation is made, for each cluster C according to the formula $\sum_{i \in C} p_i / \Delta_{cost}$, where Δ_{cost} is the difference between the expected cost of the tour, obtained by inserting the gravity center in its best position, and the cost of the tour without the gravity center.

Clusters of vertices candidate for removal are defined as follows. Let consider a tour $\{0, \ldots, j_0, i_1, \ldots, j_1, i_2, \ldots, j_2, i_3 \ldots, j_{\lambda-1}, i_0, \ldots, 0\}$ where $(j_0, i_1), (j_1, i_2), \ldots, (j_{\lambda-1}, i_0))$ are the λ highest cost edges of the tour. Then, the chains of adjacent vertices are $\{(i_1, \ldots, j_1), \ldots, ((i_{\lambda-1}, \ldots, j_{\lambda-1})\}$.

The value of a move associated with the removal of a chain Ch is measured by the ratio of saved cost over lost profit, and is computed as $\sum_{i \in Ch} p_i / \Delta_{cost}$, where Δ_{cost} is the difference between the expected cost of the tour the cost of the tour obtained by taking away the ordered subset of vertices belonging to the chain and by connecting the endpoints of the route. The results of the insertion and deletion are then compared and the best move is applied. If the best move is a deletion of a chain of nodes, then all the vertices of the chain are declared tabu for θ iterations. A random diversification mechanism is applied if the solution is not improved after a given number of iterations, by perturbing the current solution. An overall description of the tabu search algorithm is reported in Algorithm 2.

Algorithm 2: The TABU pseudo-code.

```
Input: κ,θ
Initialization: best := −∞; it = 0
Generate an initial solution s and set s_min := s
Determine the partitions
Set λ = rand[2, max(4, n/2)],
  θ = rand(5,25), κ = 5.
repeat
    Evaluate all the clusters for insertions and
      all the chains for deletions.
    Choose the best move on the basis of the
      evaluations and modify the solution s
      accordingly if the best move is a
      deletion then
        identify the tabu set T(s) forbidding
          for the next θ iterations the nodes of
          the selected chain
    end
    if it mod κ = 0 then
        apply two-opt
    end
    Evaluate the objective function value of s,
      OF(s). if s improves the previous best
      known solution then
        apply 3-opt and set best := OF(s)
    end
    if no improvement after 100 iterations
      then
        shuffle the route
    end
    it:=it+1
until a given termination criterion is met;
Return best
```

5 COMPUTATIONAL RESULTS

In this Section we discuss the numerical results obtained by applying the two algorithms. All the heuristics were implemented in Python 3 and run on a laptop with an Intel(R) 4 Core (TM) i7-4600U CPU with 8 Gb RAM and 64-bit operating system. Then, we test the performance of the TABU and the GA presented in Section 4 and identify the best solution method to be used in the more extensive numerical tests presented in the last part of the Section. We performed the experiments on 57 instances

Table 1: GA versus TABU.

Instance	Nodes	# Visited GA	# Visited TABU	%Impr
a280.tsp	280	272	151	66,77
ali535.tsp	535	534	267	64,33
berlin52.tsp	52	46	45	-5,24
bier127.tsp	127	122	112	5,40
ch130.tsp	130	125	123	4,05
ch150.tsp	150	143	129	16,62
d198.tsp	198	194	153	29,21
d493.tsp	493	492	246	77,52
d657.tsp	657	656	328	72,79
dsj1000.tsp	1000	998	500	52,72
eil101.tsp	101	92	92	-3,66
eil51.tsp	51	37	42	-19,43
eil76.tsp	76	65	65	-1,21
fl417.tsp	417	414	208	87,82
gil262.tsp	262	261	147	61,40
gr137.tsp	137	135	124	5,02
gr202.tsp	202	199	163	18,46
gr229.tsp	229	228	177	22,65
gr431.tsp	431	430	215	86,21
gr666.tsp	666	665	333	86,00
gr96.tsp	96	92	93	-1,43
kroA100.tsp	100	94	98	-0,93
kroA150.tsp	150	145	139	4,06
kroA200.tsp	200	196	130	40,77
kroB100.tsp	100	96	97	-0,62
kroB150.tsp	150	146	132	9,76
kroB200.tsp	200	197	133	43,09
kroC100.tsp	100	94	96	0,40
kroD100.tsp	100	95	97	-1,38
kroE100.tsp	100	96	95	-0,24
lin105.tsp	105	102	104	-0,58
lin318.tsp	318	316	159	74,35
linhp318.tsp	318	315	159	69,90
p654.tsp	654	653	327	79,83
pcb442.tsp	442	441	221	71,55
pr107.tsp	107	103	103	-0,13
pr124.tsp	124	123	122	-0,54
pr136.tsp	136	132	128	2,49
pr144.tsp	144	142	132	4,74
pr152.tsp	152	150	140	6,41
pr226.tsp	226	221	133	46,26
pr264.tsp	264	256	150	62,72
pr299.tsp	299	295	159	75,81
pr439.tsp	439	438	219	73,08
pr76.tsp	76	72	72	-4,86
rat195.tsp	195	191	141	31,36
rat575.tsp	575	574	287	75,99
rat783.tsp	783	782	391	78,00
rat99.tsp	99	93	95	0,40
rd100.tsp	100	94	89	0,30
rd400.tsp	400	397	200	48,80
st70.tsp	70	60	63	-0,81
ts225.tsp	225	215	136	50,19
tsp225.tsp	225	217	134	47,72
u159.tsp	159	155	144	2,94
u574.tsp	574	573	287	79,75
u724.tsp	724	723	362	74,84

taken from the TSPLIB $https://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/$ with a number of nodes ranging from 51 to 1000 nodes. To create the test instances, we considered the random cost $\tilde{c}_{ij} = \xi * \bar{c}_{ij}$, where the nominal cost over the edge (ij) has been set proportional to the Euclidean distance between node i and j and ξ is a normal random variable with given mean and variance. For both the algorithms a time limit of 60

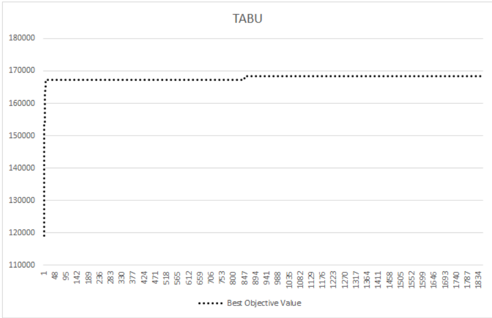

Figure 2: Iterations of the TABU.

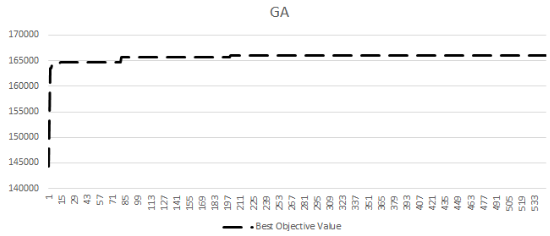

Figure 3: Iterations of the GA.

seconds has been imposed. In the Table 1, we report the name of the instances, the number of nodes, the number of nodes visited in the solutions provided by the algorithms and the percentage Improvement of GA versus TABU (column heading %Impr).

As evident, the GA is able to find better solutions than the TABU and the average improvement is around 33%. This is due to the nature of the GA which works with a a pool of complete solutions and not to the specific choice of the operators, which have not been tailored for the problem at hand. In 44 out of 57 instances, the GA finds tours visiting a higher number of nodes. When the TABU is able to outperform the GA (and a negative value of the percentage improvement is reported in the dedicated column) the number of visited nodes is equal or very close for both the methods. Analyzing more in detail the behaviour of the algorithms for a given instance (gr96.tsp) we notice, from Figures 2 and 3 that the GA performs a lower number of iterations (less than one third) than the TABU. Both algorithms are able to obtain a good solution after half of the total number of iterations, i.e. the TABU which reaches a good solution after around 900 iterations out of 1834. In Table 2 is reported the deviation of the solution values of the five random runs for the GA. The last column %Δ represents the percentage deviation between the maximum and minimum value of the objective function. This variability of the solutions arises from the choice of the random seeds, and from the randomness involved in the algorithm. This information is important to assess how robust the heuristic is in terms of the solution consistency.

Table 2: Performance of the GA with differents seeds.

Instance	**Seed 4**	**Seed 5**	**Seed 6**	**Seed 7**	**Seed 8**	%Δ
a280.tsp	5516133	5512424	5515533	5512693	5508237	0.14
ali535.tsp	22147713	22088638	22113013	22125713	22178113	0.41
berlin52.tsp	544107	544107	557231	559231	553331	2.78
bier127.tsp	65511768	65334239	64990332	65348039	65466639	0.80
ch130.tsp	3186711	3193691	3193691	3193591	3193691	0.22
ch150.tsp	4150750	4156098	4156998	4147736	4162735	0.36
d198.tsp	38627538	38640059	38631623	38644618	38638512	0.04
d493.tsp	252483148	252511548	252579548	252325790	252414367	0.10
d657.tsp	500782552	501218840	500881452	500886852	500976952	0.09
dsj1000.tsp	2.85261E+11	2.85261E+11	2.84993E+11	2.85108E+11	2.85245E+11	0.09
eil101.tsp	167737	172142	172642	172173	172928	3.09
eil51.tsp	12526	14362	14429	14516	13643	15.89
eil76.tsp	63308	63308	61345	62509	63302	3.20
fl417.tsp	104316634	104082283	104241883	104179591	104119883	0.23
gil262.tsp	4278023	4353026	4366527	4342957	4353671	2.07
gr137.tsp	682127	682227	678682	682427	682527	0.57
gr202.tsp	1089744	1090779	1091272	1090723	1090525	0.14
gr229.tsp	4415630	4415232	4416198	4416756	4413990	0.06
gr431.tsp	17558075	17555108	17557575	17556375	17556475	0.02
gr666.tsp	39501237	39493137	39515237	39488537	39497537	0.07
gr96.tsp	166044	166044	165485	166088	166013	0.36
kroA100.tsp	7941865	7904464	7793754	7874520	7931956	1.90
kroA150.tsp	21921673	22042335	22012635	21769024	21940948	1.26
kroA200.tsp	44769001	44779766	44771573	44608987	44451500	0.74
kroB100.tsp	6914730	6873487	6750118	6923030	6931382	2.69
kroB150.tsp	20240863	20307834	20116971	20019636	20297334	1.44
kroB200.tsp	35893778	35669586	35873276	35893471	35721935	0.63
kroC100.tsp	9171848	9099833	9089433	9015458	9074246	1.73
kroD100.tsp	7529474	7328771	7438351	7570674	7441974	3.30
kroE100.tsp	7264044	7211989	7161467	7290844	7265835	1.81
lin105.tsp	7106445	7055818	7106445	7075044	7101645	0.72
lin318.tsp	116486420	116785931	116720784	116892758	116846162	0.35
linhp318.tsp	113003012	113224770	113168804	113100132	113411282	0.36
p654.tsp	661050147	661458847	661284647	661080447	660779924	0.10
pcb442.tsp	221321761	221486561	221859161	221362445	221726061	0.24
pr107.tsp	25328948	25328948	25333251	25319748	25346848	0.11
pr124.tsp	46923397	46759814	46880997	47048723	46536753	1.10
pr136.tsp	53805109	53859976	53796729	53669821	53844885	0.35
pr144.tsp	56556141	56556141	56387341	56556141	56556141	0.30
pr152.tsp	84640652	84812508	84556837	84737764	84812508	0.30
pr226.tsp	204150277	204281100	204258422	203941226	204211768	0.17
pr264.tsp	146436552	146214768	146112774	145891647	146657398	0.52
pr299.tsp	151796427	152291132	152388830	152064812	151949428	0.39
pr439.tsp	595473482	597012382	596142083	596290682	596011082	0.26
pr76.tsp	19250820	18907065	19245511	19250820	19250820	1.82
rat195.tsp	2724185	2729911	2727943	2727183	2729562	0.21
rat575.tsp	43644365	43598665	43598765	43581265	43626765	0.14
rat783.tsp	92716430	92720043	92704330	92669430	92724730	0.06
rat99.tsp	386922	388434	388434	387754	388434	0.39
rd100.tsp	2236314	2236089	2224455	2163871	2225658	3.35
rd400.tsp	48288473	48256500	48547371	48542071	48941571	1.42
st70.tsp	90498	90434	90453	90434	90173	0.36
ts225.tsp	198420303	198026821	198684376	198553972	198729501	0.35
tsp225.tsp	6075338	6066911	6066092	6027446	6054097	0.79
u159.tsp	35047328	35084536	35084536	35050936	35050936	0.11
u574.tsp	272905885	272996985	272926018	272982185	272982085	0.03
u724.tsp	395250080	395429945	395306980	395237331	395381080	0.05
						1.06

From the observation of the results we notice that the relative deviation of the GA is quite small and on average around 1%. This implies that GA best fitness value fluctuates only slightly around the best solution identified by the same algorithm.

6 CONCLUSIONS

In this paper, we have presented an interesting variant of the profitable tour problem with stochastic costs under a risk.-averse perspective. We have developed two metaheuristics based on a GA and a TABU search method, respectively to solve the problem. Future work can be directed along the following directions. First, we can assume that different edges may have different correlated costs. This situation can be relevant, especially in disaster management applications.

Second, advanced heuristics in the spirit of adaptive large neighborhood search heuristics may be devised.

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